LAKE EUTROPHICATION: USING RESILIENCE EVALUATION TO COMPUTE SUSTAINABLE POLICIES

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EXTENDED ABSTRACT

We consider a sustainable management issue: How to maintain a lake in an oligotrophic state (with low input nutrients, clear water and high economic value), notwithstanding the economic interest of farmers which requires to use input nutrients? We adopt Martin [1]'s framework, in which this problem is related to a particular definition of resilience.
In a widely accepted definition, resilience is the capacity of a system to maintain some of its properties in spite of disturbance. Martin [1] proposed a precise mathematical interpretation of this concept, based on viability theory, together with methods to compute resilience values and restoration action policies. Resilience values are computed as the inverse of the cost for restoring a given interesting property, lost after a disturbance. This framework is very general, and is in principle of high interest for policy support. However, its current practical implementation is limited to problems in low dimensional space and the uncertainties on the parameters of the model are not taken into account.
In this paper, we propose a new algorithm in order to deal with problems in higher dimensional space. It uses a classification method, Support Vector Machines, which is very efficient to deal with problems in high dimensional spaces. In addition, it defines more or less cautious action policies, in order to restore the viability of a system [6]. Starting from this new development, we propose an algorithm that integrates the specificities for computing resilience values, and restoration action policies.
We apply this new approach to compute resilience values based on a model of lake eutrophication, including three parameters: the amount of phosphorus in the water, the annual phosphorus input from human activities and the amount of phosphorus in the sediments. We also include uncertainties on some parameters of the dynamical model.
The results associated with each state of the system are, in one hand, the cost for restoring the property of interest and, on the other hand, the resilience in relation to potential exogenous disturbances.
Comparing the results obtained in this paper with those in the literature, this work highlights the state areas where it is crucial to take into account the slow dynamics of the model (i.e. the amount of phosphorus in the sediments). It also emphasizes that the results are sensitive to the uncertainties on the parameters, precisely the parameters that were neglected when using the classical viability algorithm.
To conclude, the combination of the definition of resilience proposed by [1] and the new algorithm of viability introduced in this paper offers an interesting approach to sustainable development, enabling to compute resilience values and restoration action policies on more realistic models than previously.

Keywords: resilience, lake eutrophication, viability theory, SVM.

1. INTRODUCTION

The resilience of a system toward one of its properties, linked with its structure or function, is related to its ability to undergo disturbance and maintain this property [2]. Therefore, evaluating system resilience generally gives indications about how to maintain or restore, if possible, the desirable properties of the system, and thus to define sustainable management policies. In this paper, we use a recent definition of resilience [1] which directly defines management policies, and we apply it to lake eutrophication.

Lake eutrophication is a well-known example of ecosystem sudden shifts, from oligotrophic state with relatively high economic value of ecosystem services (freshwater, used for irrigation, municipal water supplies, pollution dilution, and recreation) to eutrophic state, turbid-water with poor ecosystem services value. Phosphorus (*P*) is the most critical nutrient for the eutrophication of lakes [3,4]. Excess *P* is imported to farms in the form of fertilizer and animal feed supplements. *P* is added to the soil as inorganic fertilizer or manure. Most of the *P* accumulates in soil, which may then be transported to streams and lakes during runoff events associated with snowmelt or rainstorms.

The desirable property of a lake is obviously being oligotrophic to the viewpoint of the population that benefits from its services. Nevertheless, without farming activities in its watershed, the problem of eutrophication would not have come up. Consequently, farmers have to be included in the system under consideration with their own view on the desirable property of the whole system, that is the profitabitity of their farming activites which rely on phosphorus inputs. The management issue is thus to fulfil action policies that preserve or increase the resilience of the whole system according to the twofold objective of keeping the lake in an oligotrophic state and ensuring the profitability of farmers activities.

Viability theory [5] is a suitable framework to address resilience evaluation in ecosystem models [1]. Actually, this approach deals with evolutions of the state of the system influenced by possibly several admissible controls (action policies) and governed by possibly nondeterministic dynamics (such as uncertainty on parameters, often experienced in socio-ecological models). The main concept of this theory is the viability kernel which gathers all states of the system from which there exists at least one evolution remaining in a specified constraint set in the state space. Defining the mesure of resilience as the inverse of the cost of restoration of the desirable property after a disturbance, Martin [1] showed how regarding the set of states where the desirable property holds, as a viability constraint set, allows to reach the resilience values by way of computing the viability kernel of an auxilliary problem.

However, this approach requires large computing power, especially when the state space of the system has many dimensions. Recently, new methods, based on specific learning techniques called “support vector machines” (SVMs), raised some hope to overcome some of these limitations [6,7]. This hope is suggested in particular by the universally recognised tremendous ability of SVMs to code very parsimoniously complex shapes in large dimension spaces. First experiments of the method tend to confirm the interest of this tool to approximate viability kernels.

With this new approach to compute ecosystem resilience as proposed by [1], the main contribution of this paper is to study the significance of the dynamics of P in the sediments neglected by [1] for the design of sustainable policies.

We first describe rapidly the model of lake eutrophication, then we introduce the new algorithm to compute resilience values and finally present the main results.

2. MODEL OF LAKE EUTROPHICATION

The system under consideration encompasses a lake and the farming activities in its watershed. The property of the whole system which resilience is worth evaluating is the oligotrophic state of the lake and the profitability of farming activities.

2.1. The dynamics and the controls

The model combines an ecosystem model of phosphorus dynamics and a controlled model for phosphorus input dynamics. Dynamics of phosphorus in sediments and water followed the model used by [8], with $f(P(t)) = \frac{P(t)^q}{P(t)^q + m^q}$:

$$\begin{aligned}\frac{dM}{dt} &= -kM(t) + sP(t) - rM(t)f(P(t)) \\ \frac{dP}{dt} &= -(s+h)P(t) + L(t) + rM(t)f(P(t))\end{aligned} \quad (1)$$

M is mass of phosphorus in the lake sediments and *P* is mass of phosphorus in the lake water (both described by time and with units g m^{-2}). *L* is *P* input flux (g m^{-2} y^{-1}), *s* and *h* are rate constants for sedimentation and hydrologic outflow, respectively, and *m* is the *P* mass in the water for which recycling is half of the maximum rate. Parameter *q* sets the steepness of the recycling versus *P* curve when $P \approx m$. The one dimensional model where the influence of *M* on *P* dynamics is neglected [9] is often used in literature. By comparing our results on resilience to those obtained in [1], we will show that phosphorus in the sediments has a significant impact on resilience values.

In order to show the influence of phosphorus in the sediments, we take the same dynamics as [1] for phosphorus input policies: the manager can act directly on the time variation through control *u*, with *u* bounded because modifications take time:

$$\frac{dL(t)}{dt} = u, \quad u \in [-VL_{\max}; VL_{\max}] \quad (2)$$

2.2. The property of interest regarded as state constraints

We also assume that an oligotrophic lake becomes eutrophic when the amount of phosphorus in the water increases over some fixed threshold P_{max}. Consequently, the lake ecosystem objective is reached when the positive variable *P* satisfies:

$$P \in [0; P_{\max}] \quad (3)$$

We suppose that farmers' benefit depends linearly on the inputs of phosphorus. Consequently, profitability is reached when the value of phosphorus inputs is higher than a given threshold L_{min} and lower than the maximal legal value L_{max}:

$$L \in [L\min; L\max] \quad (4)$$

Equations 1, 2, 3 and 4 can be written synthetically under the formalism of viability theory with x(t): = (L(t),P(t), M(t)):

$$x'(t) = \begin{pmatrix} L'(t) \\ P'(t) \\ M'(t) \end{pmatrix} = \begin{pmatrix} u(t) \\ -(s+h)P(t) + L(t) + rM(t)f(P(t)) \\ -kM(t) + sP(t) - rM(t)f(P(t)) \end{pmatrix} (5)$$

under the constraints $K = [L_{\min}; L_{\max}] \times [0; P_{\max}] \times [0; +\infty]$ (6)

2.3. The cost functions

We recall that these cost functions are used to evaluate the resilience of the property defined by the set *K* which, in this application, ensures the profitability of the farmers' activities and keeps the lake in an oligotrophic state. Consequently, these functions have to satisfy two conditions: first, the cost of a trajectory along which the property is maintained is null; second, the cost of a trajectory such that the property cannot be restored is infinite. Furthermore, the trajectory starting at *x* with minimal cost is the best trajectory to follow according to the objective to maintain or at least restore the property of interest before time *T*. In practice, the ways of evaluating the cost of an evolution *x(.) = (L(.),P(.), M(.))* satisfying equation 5 are numerous and depend on the situation. We use a cost function made up of three weighted terms: the first term, which corresponds to the ecological cost, is the time spent in an eutrophic state; the second one, which is an economic cost, measures the time duration of the period of negative profits weighted by the norm of these negative profits; the third one corresponds to the management cautiouness cost: when the state of the system does not belong to the viability kernel of *K*,*Viab(K)*, policies have to be strictly fulfiled to restore, if possible, the desired property with minimal cost. The function that associates *x* with the minimal cost over all trajectories starting at *x* is then λ defined by:

$$\lambda_K(x) = \min_{x(.)}(c_3\int \chi_{P \geq P\max}(x(\tau))d\tau + c_2\int (L\min - L)\chi_{L \leq L\min}(x(\tau))d\tau + c_1\int \chi_{x \notin Viab(K)}(x(\tau))d\tau) \quad (7)$$

with $\chi_{P \geq P\max}(x) = 1$ if $P \geq P\max$, $\chi_{L \leq L\min}(x) = 1$ if $L \leq L\min$ and $\chi_{x \notin Viab(K)} = 1$ if the state of the system does not belong to *Viab(K)*, and 0 otherwise.

2.4. The disturbances

In this simple example, we consider disturbances $D\alpha$ corresponding to a sudden increase of the concentration of phosphorus in the lake. α represents the maximal intensity of the anticipated disturbance. If a disturbance $D\alpha$ occurs when the state of the system is *x*, it will jump to a state y belonging to set $D\alpha$ *(x)*:

$$D\alpha(x) := [x; x + (0; \alpha, 0)] \quad (8)$$

3. METHODS

3.1. Resilience measure

The measure of the system resilience a state *x* toward one of its properties (represented by the set *K* in the state space) facing anticipated disturbances *D*, is the inverse of the maximal cost of restoration over all jumps from *x* described by *D*. In this simple anticipated disturbance model, the sudden increase of *P* with maximal intensity α produces the maximal cost of restoration. Then:

$$R_{K,D}(x) = 1/(\lambda_K(x + (0;\alpha;0)) \quad (9)$$

3.2. Resilience value computation

The tricky point in resilience evaluation is the cost function computation.
The viability kernel of *K*, *Viab(K)*, gathers all states from which there exists at least one evolution remaining in *K*. Therefore, the viability kernel and the set of state with null restoration cost merge. Saint-Pierre's viability algorithm [10] can also be used to compute the whole resilience values but it requires one additional dimension representing the cost. However, viability algorithms face the dimensionality curse: the computation cost grows exponentially with the dimensionality of the state space. New developments [6] introduce a classification method, Support Vector Machines [11,12], which is very efficient to deal with problems of high dimension. But, in the case of resilience, it still requires to solve a problem with one added artificial dimension.
Starting from [6], we develop a new algorithm that integrates the specificities for computing resilience values, taking into account the uncertainties on parameters.

3.3. Algorithm to compute resilience values

The main advantage of this new algorithm is that it can work without adding a specific dimension to compute the restoration cost. It also offers facilities to take into account the uncertainties on some parameters, very often encountered in ecological modelling. The algorithm involves two steps: approximation of the viability kernel of the system with uncertainties and computation of resilience values.
It is straightforward to adapt the SVM viability kernel approximation algorithm in order to introduce uncertainties on a given parameter $a \in [a\min; a\max]$. We discretise the interval

and test all these discrete values for each state: if the state is non-viable for at least one value of these values, the state is non-viable.
To compute cost function values, we start from the system approximated viability kernel and we gradually add the points that can reach the viability kernel with cost $\lambda < n \times d\lambda$ ($d\lambda$ being the cost step). In this case, we fix the cost variation at each iteration and we infer the time step for computing the state: $dt^* = \lambda / d\lambda$.
Let consider a grid H_h of points x_h, covering the set H representing the range of the system state space under study, including the subset of states showing the property of interest $K \subset H$. At iteration *n*, we define the set C_n including all the states of H_h for which there exists a control function leading the system into the viability kernel with a cost $\lambda < n \times d\lambda$. This discrete set is used to define a continuous one, $L(C_n)$, by training a SVM on a learning set *S*, composed of the points of C_n (labelled +1) and the other points of H_h (labelled -1). This procedure provides the contours of the cost function with increment *dλ*.
Besides the previously mentioned properties, SVM is also interesting aspect because it provides a kind of barrier function on the boundary of the viability kernel and thus allows to use optimisation techniques to find a viable control u^*, with respect to $L(C_n)$ (see [6] for details). The state $x_h^*(t + dt^*) = x_h + \varphi(x_h, u^*)dt^*$ defines the position the most inside $L(C_n)$ among all the possibilities. Algorithm 1 sums up the main steps of the procedure.

Init Compute the viability kernel of the system, define it as $L(C_0)$; $n = -1$
Repeat
    $S \leftarrow$ empty set ; $n = n + 1$; $C_n \leftarrow$ empty set
    For all $x_h \in H_h$
        If $x_h \in L(C_n)$ or $x_h^*(t + dt^*) \in L(C_n)$
            $S \leftarrow S \bigcup (x_h;+1)$; $C_n \leftarrow C_n \bigcup x_h$
        Else
            $S \leftarrow S \bigcup (x_h;-1)$
    End for
    Compute $L(C_{n+1})$ from S
While $C_{n+1} = C_n$

Algorithm1: SVM resilience approximation algorithm

4. RESULTS

4.1. Viability kernels

The aim is to define the levels of phosphorus in the water (*P*), in the sediments (*M*) and the inputs in the water (*L*) that are compatible with the objective to maintain the lake in an oligotrophic state, while ensuring profitability for the farmers. To achieve this aim, we approximate the viability kernel of the system, first using the algorithm described in [6] and then with the algorithm described in this paper, taking into account the uncertainties on parameter m ($m \in [0.8;1.2]$). Because the value of *m* is estimated from data for each lake, it cannot be known with certainty and we take into account its variability.
The viability kernel (*Viab*(*K*)) coloured in blue gathers the viable states, where it is possible to keep the system inside the set of interesting states *K*. The set *K\Viab(K)* contains the situations where, whatever the actions applied, the system will go out from *K*. On the graph, we show with a dotted line an example of a trajectory starting from a non-viable point and satisfying $u = -VL\max$: even with the maximal decreasing rate of phosphorus inputs, the objective is not met and the lake is doomed to become eutrophic.

The projections onto the (L,P)-plan vary very slightly with M and is very close to the viability kernel computed in [1]: consequently, with such model parameters, the concentration of phosphorus in the sediments doesn't affect the viability kernel shape. If we include the uncertainties, the approximation of the viability kernel doesn't change.

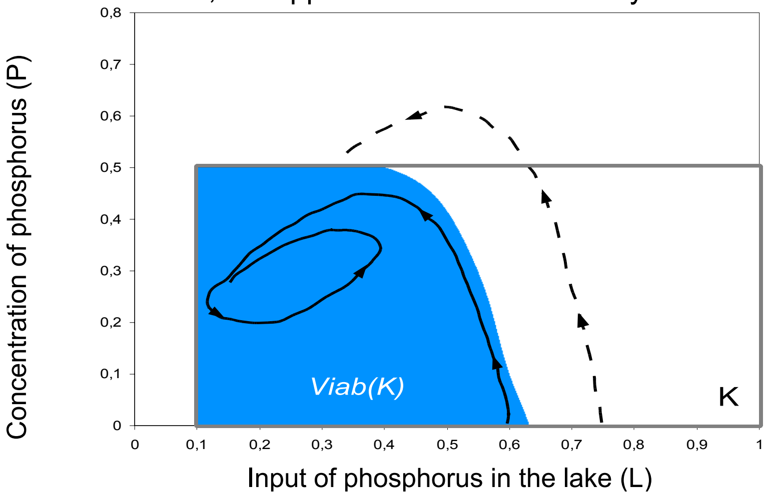

Figure 1: Projection of the approximation of viability kernel for P_{max}=0.5. The trajectory in dotted line starts from a non-viable point and the one in solid one from a viable point. K is represented by the black rectangle. The maximal velocity of phosphorus input variations is $VL\max = 0.1$. The viability kernel is colored in blue.

We now approximate the viability kernel for P_{max}=1.2 (figure 2). The results highlight the crucial role of the slow dynamics in the model: the higher the amount of phosphorus in the sediments is, the smaller is the viability domain.

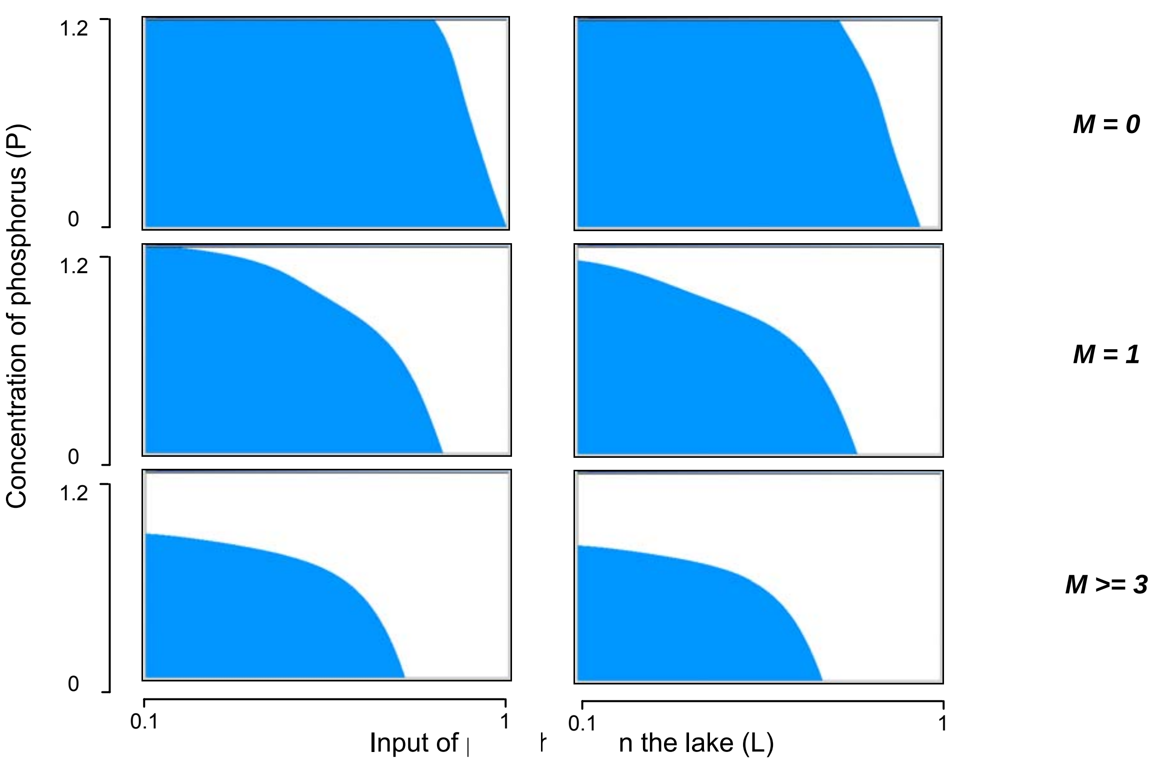

Figure 2: Projection of the approximation of viability kernel for P_{max}=1.2.

In all the configurations, there are situations for which the property of interest cannot be maintained. If we take into account the uncertainties, the viability kernel changes, especially for the smallest values of M. For $M < 3$, the viability kernel is smaller than the one without uncertainties and the objective can be maintained in fewer situations. For $M \geq 3$, the uncertainties have no influence. The next issue is then to determine if the system (including uncertainties) can be restored and if yes, at what price.

4.2. Restoration costs

The viability kernel is the 0-level of the cost function. Starting from a non viable state, the system is doomed to leave the viability constraint set *K*. But, in some cases, it is possible to restore the property of interest. We evaluate the non–null values of the cost function using the algorithm described in this paper. Figure 3 presents the projection, onto the (*L*,*P*)-plane for different values of *M*, of the values of the cost of restoration for P_{max}=1.2, including the uncertainties on parameter *m.*

The results associated which each state of the system are the costs for restoring the property of interest: for all states, the cost is finite and the system can be thus restored. These results highlight the fact that the cost is also sensitive to the values of *M*.

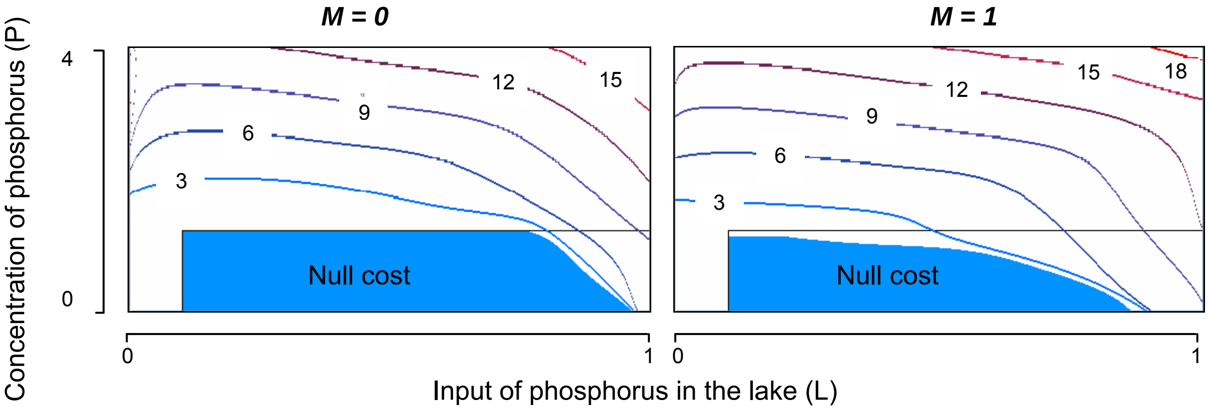

Figure 3: Projection of the approximation of the cost values for P_{max}=1.2. c_1=1, c_2=3 and c_3=10.

4.3. Resilience values

The resilience is the inverse of the cost associated with the effort to restore the property of interest, lost due to potential exogenous disturbances. We suppose that, whatever the state x of the system, the maximal disturbance cost is caused by a jump of magnitude $\alpha = 0.5$. We evaluate the resilience for each value of the cost function, for P_{max}=1.2 and taking into account the uncertainties on the parameters (figure 4).

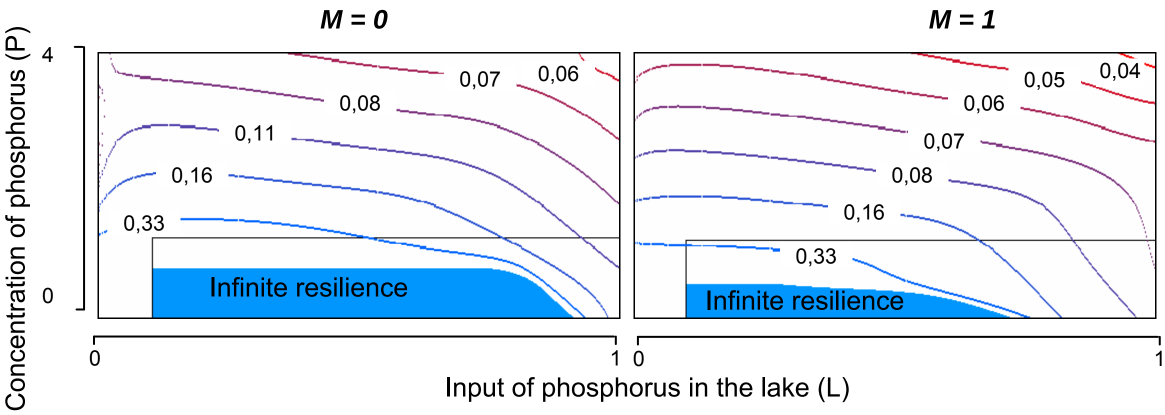

Figure 4: Projection of the approximation of the resilience values for P_{max}=1.2.

Two areas are discriminated:

- In blue, the set of states of infinite resilience (or resilient states): there exists at least one control function that allows maintaining the property of interest;

- Between the levels curves, the set of finite resilience: the oligotrophic state and profitability for the farmers can be restored.

5. CONCLUSIONS

Resilience can be defined inside the formalism of viability theory. But current algorithms to compute viability kernel and resilience values are operational only for problem of low dimension. We propose a new algorithm, based on SVMs, to compute resilience values that is more parsimonious and enhances the potential of the approach to evaluate resilience in usual ecosystem models. Moreover, it can easily take into account some uncertainties on parameters, which is an asset in ecological modelling.
We illustrated this approach with a model of lake eutrophication in 3 dimensions. Once defined a desirable property, a cost function associated with its restoration and anticipated disturbances, we computed the resilience values for each state of the system. We emphasized the role of the slow dynamics (the sediments) on the results of the analysis.
In the future, it will be interesting to consider an even more realistic model of lake eutrophication, including the dynamics of the phosphorus in the soil. In addition, when the number of parameters with uncertainties is important, it is difficult to use this algorithm because the uncertainties are discretised. The next step is thus to improve the procedure in order to overcome this limitation.

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