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The viability theory to control complex food processes

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Abstract The viability theory developed by [Aubin(1991)] has been adapted to calculate the optimal monitoring of cheese ripening process. This method was used efficiently in ecology or in finance but never in food process. The aim was to find the controls allowing reaching a compromise between the quality of the ripened cheeses and the production costs. A viability kernel and the costs simulations of the viable trajectories were computed. Then, the optimal ripening trajectories were validated during pilot trials. The results were finally compared to those obtained for cheeses ripened at 92% of relative humidity and 12°C of temperature, the conditions usually applied in dairy industries for the Camembert ripening process.

Keywords Simulation, optimization, viability theory, food processing, cheese ripening.

1 Viability theory to optimize complex food processes

Most of food processes such as cheese ripening are particularly difficult to understand. They comprise multiple levels of organizations which depend on local interactions. The relationships across these levels are not obvious, for example between microbiological, physicochemical, biochemical and sensory levels. Optimizing the control of such complex systems is not an easy task. The response surface methodology is one of the most frequent optimization procedures in food science. However, this approach is limited and during the last decades optimization methods based on food model have been developed. In this context, we implemented a new method based on a food process model and the viability theory to control the Camembert cheese ripening. The viability theory has ever been applied in sociology [Bonneuil(2000)], ecology [Bonneuil and Mullers(1997)], [Martin(2004)] or in finance, never in food science. Our aim was to optimize the control

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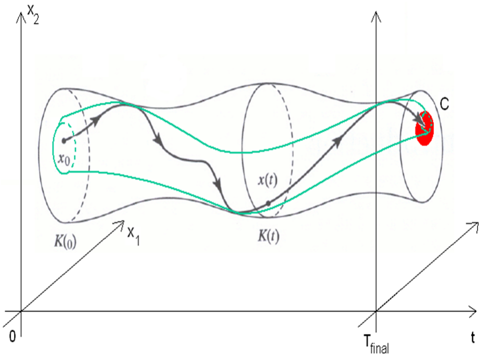


Fig. 1 Oversimplified drawing of a constraints domain (in black), viability kernel (in green), and viable trajectory (black arrows) reaching the target C (in red).

of cheese ripening while minimizing operating cost and maintaining the product quality at an acceptable level.

The viability theory [Aubin(1991)] aims at controlling dynamical systems x'(t)=f(x(t),u(t)) with the goal to maintain them inside a given constrained set K (Equation 1).

$$Viab_F(K) := \{x \in K \mid \exists x(\cdot) \in S_F(x), \forall t > 0, x(t) \in K\} \quad (1)$$

Such problems are frequent in biology, where the systems die or badly deteriorate when they leave some regions of the state space. This method analyzes the compatibility between the dynamics and the state constraints. It also determines the set of controls that would prevent the system from violating the state constraints. According to the application, the time scale of interest may be finite: we aim at maintaining the state of the system in the constrained K during a process of duration T (finite) for instance. If one trajectory starting at state x(0), remains in K for t in [0,T], it means that there exists a control function, such that the property holds during the time scale of interest, and x(0) is called a viable point. The main concept of viability theory is the viability kernel, denoted ViabF(k), which gathers all states from which there exists at least one control function u(t) such that the state of the system x(t) remains in K for t in [0,T].

If we denote SF(x), the set of evolutions governed by the controlled dynamical system x'(t)=f(x(t),u(t)), the viability kernel is defined by (Equation 2):

$$Viab_F(K) := \{x \in K \mid \exists x(\cdot) \in S_F(x), \forall r \in [0,T], x(t) \in K\} \quad (2)$$

The "viability kernel" is the subset of K that contains the equilibriums and the states from which there exists at least one trajectory remaining in K.

A cost function can also be used to associate a trajectory x(.) with its cost. The aim is to reach a target (C) with an optimal trajectory (minimal cost). It is a variant of the viability problem (Equation 3) called capture basin.

$$Capt_F(K,C) = \{x \in K \mid \exists x(\cdot) \in S_F(x), \exists t* > 0, x(t*) \in C, \forall t \in [0,t*], x(t) \in K\} \quad (3)$$

The Figure 1 is an oversimplified scheme of viability kernel with a trajectory reaching a target (C).

Numerical schemes to solve 'viability' or 'capture' problems were proposed by [Saint-Pierre(1994)]: viability kernel algorithm computes, for a given grid Gh, a discrete viability kernel that converges to the viability kernel ViabF(K) when the grid resolution h tends toward 0. It is the approach used in this work.

2 Application to Camembert cheese ripening process

2.1 A mechanistic cheese mass loss model

In this study the application has been developed to optimize the Camembert cheese mass loss during a ripening process. The problem was to find the best compromise between a good quality of cheese at the end of the process and low productions costs (few control variations, shorter ripening...)

The evolution of the cheese mass loss was considered as governed by the dynamics described in (4) and (5) [Helias et al.(2007)Helias, Mirade, and Corrieu].

$$\frac{dm}{dt} = s\left\{w_{O2}.r_{O_2} - w_{CO2}.r_{CO_2} - k\left[a_w.p_{sv}(T_s) - rh.p_{sv}(T_\infty)\right]\right\} \tag{4}$$

$$\frac{dT_S}{dt} = \frac{s}{m.C}\left\{h(T_\infty - T_s) + \varepsilon\sigma(T_\infty^4 - T_s^4) - \lambda k\left[a_w.p_{sv}(T_s) - rh.p_{sv}(T_\infty)\right] + \alpha\frac{r_{O_2} + r_{CO_2}}{2}\right\} \tag{5}$$

The evolution of the cheese ripening mass loss was considered as governed by the dynamic described in (4) and (5) [Helias et al.(2007)Helias, Mirade, and Corrieu].

The cheese mass loss during ripening is linked to the evaporation phenomena and the carbon loss through respiration of microorganisms (Equation 4). The evaporation increases with lower relative humidity in the ripening chamber and higher temperature on the cheese surface. This temperature (Equation 5) changes with the ripening chamber temperature, evaporation phenomena and respiration of microorganisms. The respiration increases the cheese surface temperature because heat is produced with the substrate degradation. In those equations, t represents the time, m is the cheese mass (kg), Ts the temperature at the cheese surface ($Kelvin$), r_{O_2} rate of the oxygen consumption ($mol.m^{-2}.s^{-1}$), r_{CO_2} rate of the dioxyde production($mol.m^{-2}.s^{-1}$), rh the relative humidity (expressed between 0 and 1) and T_∞ the temperature in the ripening room ($Kelvin$). The parameters $w_{o2}$and $w_{co2}$are molar masses ($kg.mol^{-1}$), s is the cheese surface (m^2), a_w is the cheese surface water activity ($dimensionless$), p_{sv} is the saturation vapor pressure (Pa), k is the average water transfer coefficient ($kg.m^{-2}.Pa^{-1}.s^{-1}$), C is the cheese specific heat ($J.kg^{-1}.K^{-1}$), h is the average convective heat transfer coefficient ($W.m^{-2}.K^{-1}$), ε is the cheese emissivity ($dimensionless$), σ is the Stefan-Boltzmann constant ($W.m^{-2}.K^{-4}$), α is the respiration heat of for 1 mol of carbon dioxide release ($J.mol^{-1}$) and λ is the latent vaporization heat of water ($J.kg^{-1}$). This model was developed and validated on experimental data sets with a relative error between 1.9-3.2%.

So, the decision variables considered in this model are the relative humidity (85%-98%) and temperature (8°C-16°C). The state variables are the cheese mass, the cheese surface temperature, and the micro-organisms respiration.

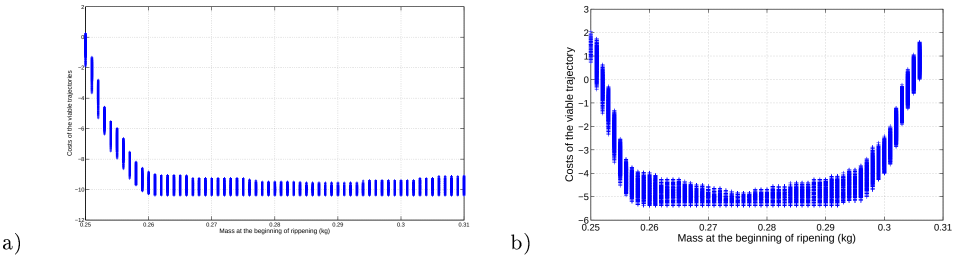


Fig. 2 : Ripening cost simulation associated with the possible cheese weight (kg) at the beginning of the process (first day of ripening). The simulation is in case of 15 days of ripening in the case (a) and 8 days of ripening in (b).

2.2 Viability kernel of cheese mass loss during ripening

Then, a viability kernel was computed based on this cheese mass loss model. The constraint to built the kernel were, first, constraint to reach a target of cheese mass (250g-270g) at the end of the 14 days ripening process. The second constraint was to obtain a gas (CO_2, O_2) rate specific evolution during the process which attests the quality of the control as regard to its impact on the micro-organisms “full capacities”. This expected respiration evolution was extracted from expert knowledge. The respiration rate should begin at level 0 on day 1 (microbial growth latency), reach a maximum between day 3 and day 8 and decrease slowly during the last days of ripening. First step consisted in checking if it exists one control couple (rh, T_∞) for every $(m_x, Ts_x, r_{o_2x}, r_{co_2x}, t)$ at time t such as $(m_y, Ts_y, r_{O_2y}, r_{CO_2y}, t+1)$ belonged to the viability kernel at time t + 1 . Then, each points fo which at least one solution exists is considered as viable.

2.3 Ripening control optimization

At the same time, the costs of the trajectories were calculated. The cost function was composed of changing control costs and trajectory robustness cost. The robustness of a trajectory was represented by the number of possible controls (remain in the viability kernel) on every points of the trajectory. This calculation requires an exhaustive search in the control space at each time step. This method suffers from the dimensionality curse. The calculation time has been estimated at 5 month. In our study, the computation was made possible by the distribution of the algorithm on a cluster composed of 200 CPU (Central Processing Unit). The calculation time was reduced at 4 days. The results of the total costs for the trajectories are presented in Figure 2, they represent the possible mass and their associated cost for 15 days of ripening (a) and for 8 days of ripening (b).

Simulations for shorter ripening have been computed. For 8 days of ripening, there is no viable trajectory for cheese with a weight higher than 317g (see Figure 4(b)). The costs were calculated for ripening trajectories finishing at 8, to 15 days. The duration was then added to the total cost for each trajectory (a shorter ripening is less expensive for a dairy factory). The total mass loss during the process was also included in the total cost function. Then the trajectories with the lower costs were considered and some optimal ones were chosen. The approach is resumed in Figure 3.

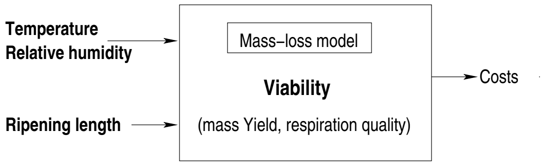
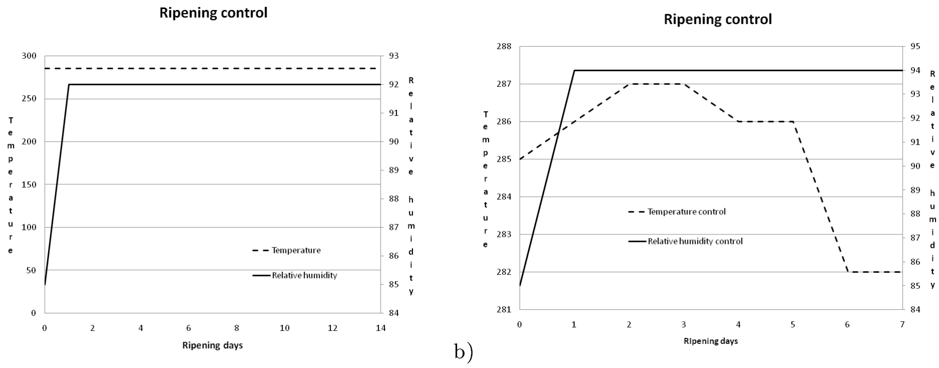


Fig. 3 Optimization of the ripening process approach



a) b)

Fig. 4 (a) Ripening control used to apply in dairy factories and (b) optimal ripening control found with the method based on the viability theory. Relative Humidity control is represented by the continuous line and the temperature control by the dotted line.

The simulation depends on the controls and the ripening length choosen. Then, the viability is carried out through the mass loss constraint and the respiration quality constraint on the bases of the mass loss model [Helias et al.(2007)Helias, Mirade, and Corrieu]. Finally, the costs of each ripening trajectories were obtained.

We found one of the best trajectories for a cheese of 280g and a ripening of 8 days long. This trajectory was then validated during an experimental Camembert-cheese ripening in a pilot. Every day of ripening, the controls were fixed at the level given by the simulation results (Figure 4 (b)).

The beginning of ripening is always the same: 85% of relative humidity and 12°C for temperature. This first step of ripening is necessary to curd drying allowing micro-organisms growth on the cheese surface. Then, the optimal trajectory differs from the classical one. The relative humidity is higher 94% instead of 92% and the control of temperature is modified instead of staying the same. The temperature control is successively 12°C, 13°C, 14°C-14°C, 12°C-12°C, and 9°C before wrapping the cheese.

During, the pilot trial the effect of the temperature and relative humidity "optimal" control was evaluated on several ripening kinetics: respiratory activity, mass loss, sensory evolution... Temperature, relative humidity and gas concentration were measured continuously during the ripening.

The results of the optimal trajectory were finally compared with those obtained in a classical ripening control applied in dairy industry (92% of relative humidity and 12°C during 15 days) applied during a second pilot trial. Cheese mass loss evolutions and respiration evolutions during the trials were compared. The sensory properties between both cheeses groups, the first obtained with optimal control and the second with usual ripening condition were also analyzed. Five sensory indicators were evaluated: cheese surface humidity, cheese color, the under-rind size and for the two cheese faces: cheese *Penicillium camemberti* mycelia growth and thickness of the mycelia... The mycelia is the white cover typical of Camembert-cheese. The sensory profile at day 15 for the optimized ripening control was the same that the profile at day 20 of the classical control. The optimized ripening control application in a pilot trial give interesting results with a ripening shortened of 7 days and a product with a good quality level for consumer 5 days before the cheese ripened following classical controls (92% RH, 12°C).

3 Conclusion

Viability theory framework was successfully applied to cheese ripening process. Now it would be interesting to go further by trying to optimize other food processes. Further work is also in progress to better take into account the robustness of the trajectories, with the computation of the distance to the viability kernel boundary (it is a measure of the robustness of a state). This method will also be useful to take into account the uncertainty of the decision variables.

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