A viability approach to control food processes: Application to a Camembert cheese ripening process.

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Abstract

This paper addresses the issue of studying the viability theory, developed for model exploration purposes and in our example applied to the optimisation of a food operation. The aim is to identify the whole set of viable trajectories for a given process. It focuses on the preservation of some specific properties of the system (constraints in the state space).On the basis of this set, a set of actions is identified and robustness is discussed. The proposed framework was adapted to a Camembert ripening model to identify the subset of the space state where almost one evolution starting in the subset remains indefinitely inside of the domain of some viability constraints, that makes it possible to reach a predefined quality target. The results were applied at the pilot scale and are discussed in this paper. The cheese ripening process was shortened by four days without significant changes in the micro-organisms kinetics and a good sensory quality of the cheese.

Keywords: knowledge integration, viability theory, food processing, control, cheese ripening

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Nomenclature

t	time	(s)
m	cheese mass	(kg)
T_s	cheese surface temperature	(Kelvin)
r_{o2}	dioxygen consumption rate	$(mol.m^{-2}.s^{-1})$
r_{co2}	carbon dioxyde consumption rate	$(mol.m^{-2}.s^{-1})$
rh	ripening room relative humidity	(%)
T_∞	ripening room temperature	(Kelvin)
w_{02}	dioxygen molar mass	$(kg.mol^{-1})$
w_{co2}	carbon dioxyde molar mass	$(kg.mol^{-1})$
s	cheese surface	(m^2)
S	a set of trajectories	
k	the set of constraints	
C	the target to be reached	
x	the vector space of state variables	
SRT	Standard ripening trajectory	
T	the finite time where the target is	days
TVA	Viable ripening trajectory	

1. Introduction

The cheese ripening process, such as the one used for Camembert, is considered to be a complex system. Numerous interactions take place at different levels of scale, from microscopic to macroscopic level, over time. To enhance camembert ripening control, numerous studies have been carried out in the food sciences, but there is still lack of knowledge. Despite the number of experimental databases collected, they remain incomplete, and it is obviously impossible to carry out all of the variable combinations through experimental trials because of the time necessary (41 days per trial). However, models have been developed to help us to more effectively understand such complex processes (Perrot et al., 2011). Cheese processing has been modelled by means of mechanistic models (Riahi et al., 2007), the partial least square method (Cabezas et al., 2006), neuronal methods (Jimenez-Marquez et al., 2003), dynamic Bayesian networks (Baudrit et al., 2008), genetic algorithms (Barriere et al., 2008), stochastic models (Aziza et al., 2006), finite element methods (Bona et al., 2007) and the fuzzy symbolic approach (Perrot, 2004,Ioannou et al., 2003). Simulations can be performed with these models to investigate food processes and to better understand them.

The aim of this study was to adapt a viability approach, for control purposes. For study the dynamics of the process with a viability theory (VT) point of view (Aubin et al., 2005), the variables and constraints are characterized by the geometry that its generates in the state space of the model, then the space is classified to identify, for example, the viability kernel : the subset of the space where almost one evolution starting in the subset remains indefinitely inside of the domain of some (viability) constraints.A fundamental difference between VT and classic control engineering, is that VT represents a deep comprehension of the behavioral space, replacing the update procedure from single-valued maps to set-valued maps. In VT, state and control variables theoretically belong to n-dimensional vector spaces that allows to study the influence of procedures on several controls on the process (Aubin, 1991). For the end user, its knowledge offer a freedom of choice to incorporate new criteria in the decision process. For the decision support system, VT offer an unique oportunity to connect the set structure of the model with an evolutionary optimization mechanism. In control and optimization, the dimension of the problem structure is the first bottleneck for problem solving, it generally define the limits of the application because the curse of dimensionality (CoD). With the benefits of distributed computing environments, it is possibile to avoid the CoD without loss of fundamental characteristics of the model (Reuillon et al., 2008), as is required for food processing.This theory has been applied to ecological problems by Bonneuil and Mullers (1997). It was also applied to the renewable resource domain, for example, to the viability of trophic interactions in a marine ecosystem (Chapel et al., 2008) or to the restoration cost of a eutrophic lake (Martin, 2004). Other applications can also be found in the areas of finance (Bonneuil, 2004), highway traffic fluxes (Aubin et al., 2005) and sociology Bonneuil (2000). This is the first time that the viability theory has been applied to food processes. It is applied on the cheese ripening process. It relies on a mathematical development coupling the viability theory developed by Aubin (1991), a high performance computing and a robustness evaluation of the whole viable trajectories extract from expertise handling.

The work is presented in **third parts**. **The first part** is dedicated to the theoretical framework of the viability theory. The main concepts of the viability theory are defined in Section 2.1. **The second part** presents the food model treated as example: the cheese ripening model. In section 2.2 this model is presented. The adaptation of the viability concept to the problem of cheese ripening including algorithm and computation is detailed section 2.3 to 2.4. The section 2.5 presents the cheese ripening trials used to test the algorithm. The viability set and the robust trajectory results are presented in **the third part**, section 3. In this section, we also describe the test of one of these trajectories during an experimental trial, in comparison to a ripening processed under standard conditions.

2. Material and Methods

2.1. The viability theory

The viability theory of Aubin (1991) aims at controlling dynamical systems that focus on the preservation of certain specific properties of the system (constraints in the state space).

Let $X \subset \mathbb{R}^n$ be the state space of the system. This system state evolves over time $x(.) : t \to x(t) \in X$ for $t \in R_+ := [0, +\infty[$. We assume that its evolution depends on the state of the system as well as controls. It is governed by a control dynamical system:

$$\begin{cases} x'(t) = f(x(t), u(t)) \qquad (action) \\ u(t) \in U(x(t)) \qquad (retroaction) \end{cases} \tag{1}$$

where the available controls u at time t belong to the set $U(x(t)) \subset \mathbb{R}^p$. A solution for this system is a trajectory $t \to x(t)$ so that a measurable control function $t \to u(t)$ exists so that conditions (1) are satisfied for almost all t.

Viability constraints are described by a closed subset$K \subset X$ of the state space. They describe the viability of the system since the state of the system is no longer viable outside of K .

2.1.1. Viability kernel

The general definition of the basis of the viability theory is the viability kernel, referred to as $Viab_{f,U}(K)$, which contains all states from which at least one control function $u(t)$ exists so that the state of the system $x(t)$ remains in K for t in $[0, T]$. We recall that

$S_{f,U}(x)$ is the set of all trajectories governed by the controlled dynamical system (1) starting from x. The viability kernel is then defined by Equation 2:

$$Viab_{f,U}(K) := \{x \in K \mid \exists x(\cdot) \in S_{f,U}(x), \forall t \in [0,T],\ x(t) \in K\} \tag{2}$$

This viability kernel also determines the set of controls that would prevent the system from violating the state constraints. The particular case of the capture basin is to find trajectories remaining in the constraint domain that reach a target C within a finite time. This is a variant of the viability problem (Equation 3)known as capture basin $Capt_{f,U}(K,C)$.

$$Capt_{f,U}(K,C) = \{x \in K \mid \exists x(\cdot) \in S_{f,U}(x), \exists t* > 0,\ x(t*) \in C, \forall t \in [0,t*], x(t) \in K\} \tag{3}$$

$t^*$is the time at which the target is reached. The trajectory $x(.)$ must remain in the constraint set K before reaching the target C. For our application,the target C is the Camembert characteristic to be reached. For example cheese mass must be at least be of 0.25 kg (defined by the protected designation of origin law).

2.2. The Camembert ripening model

The evolution of Camembert ripening was considered to be governed by cheese mass loss dynamics, including microorganism respiration described in Equations (4) and (5) Helias et al. (2007).

$$\frac{dm}{dt} = s\left\{w_{o_2}.r_{o_2} - w_{co_2}.r_{co_2} - k\left[a_w.p_{sv}(T_s) - rh.p_{sv}(T_\infty)\right]\right\} \tag{4}$$

$$\frac{dT_S}{dt} = \frac{s}{m.C}\left\{h(T_\infty - T_s) + \varepsilon\sigma(T_\infty^4 - T_s^4) - \lambda k\left[a_w.p_{sv}(T_s) - rh.p_{sv}(T_\infty)\right] + \alpha\frac{r_{o_2} + r_{co_2}}{2}\right\} \tag{5}$$

In these equations, t represents the time, m the cheese mass (kg), T_s the temperature at the cheese surface ($Kelvin$), r_{O_2} the oxygen consumption rate ($mol.m^{-2}.s^{-1}$), r_{CO_2} the dioxyde production rate ($mol.m^{-2}.s^{-1}$), rh the relative humidity (expressed between 0 and 1) and T_∞ the temperature in the ripening room ($Kelvin$). The parameters $w_{o_2}$and $w_{co_2}$are molar masses ($kg.mol^{-1}$), s is the cheese surface (m^2), a_w is the cheese surface water activity ($dimensionless$), p_{sv} is the saturation vapor pressure (Pa), k is the average water transfer coefficient ($kg.m^{-2}.Pa^{-1}.s^{-1}$), C is the cheese specific heat ($J.kg^{-1}.K^{-1}$), h is the average convective heat transfer coefficient ($W.m^{-2}.K^{-1}$), ε is the cheese emissivity ($dimensionless$), σ is the Stefan-Boltzmann constant ($W.m^{-2}.K^{-4}$), α is the respiration heat for 1 mol of carbon dioxide release ($J.mol^{-1}$) and λ is the latent vaporization heat of water ($J.kg^{-1}$).This model was developed and validated on experimental data sets with a relative error between 1.9-3.2%. In order to be able to use this existing model for simulation, the model was modified so that the gas composition was no longer measured online but was instead extrapolated from experimental curves of microorganism respiration during ripening at 281 K, 285 K and 289 K and at 92% relative humidity.

This empirical respiration model coupled to the existing mass loss model, induces uncertainty in the prediction. The aim was to test the viability theory for this commonly encountered case because model generalisation is rarely perfect.

Finally, for the viability study, the space dimension is 5. The control variables considered are relative humidity and temperature. The state variables are the cheese mass, cheese surface temperature and respiration r_{co_2}($r_{o_2}$is deduced from $r_{co_2}$with the assumption of equimolarity Helias et al., 2007).

2.3. Determining the viability kernel for camembert cheese ripening process : algorithm and computation

Numerical schemes to solve 'viability' or 'capture' problems were proposed by Saint-Pierre (1994): for a given time step Δt and a given grid G_h in the state space, the viability kernel algorithm computes a discrete viability kernel that converges to the viability kernel $Viab_{f,u}(K)$ when the time step and the grid resolution tend toward 0. This is the approach used in this work, the ripening model was discretised over time using a Euler scheme. Moreover, the state space, the control space, the constraints and the target were discretised on regular grids. State and control spaces, constraints set, targets for each variables, size of the grid linked to variables discretization represent the parametrization of the algorithm. They have an influence on the results. As regard to the computing complexity of such an approach, we have chosen to fix those parameters by integration of the existing expert knowledge. Description of all of those parameters are presented below.

2.3.1. The constraint set

The vector space X consists of three state variables: cheese mass, cheese surface temperature and respiration level (see Section 2.2). The constraints set is a subset of this three dimensional space. The bound values stem from the experimental limits, the legal norms and expert interviews presented in table 1. The cheese surface temperature is an intermediate variable necessary to allow cheese mass loss calculus, at each time step, using the camembert model. Nevertheless it is not considered for viability results analysis and is not an issue for the experts. Constraints and target on this variable represent the maximal range given by the experts for the ripening room temperature and are not limitant. The two other variables, respiration rate and mass loss are the key variables. A good ripening control should be a compromise to ensure a good behavior for those two state variables. Indeed if some high humidity of the air could be a good answer to the problem of limiting cheese mass loss, it is not always a good one for the microorganisms growth (Sicard et al., 2011b) and a compromise should be found using also the temperature as a lever.

[Table 1 around here]

In this sense,one of the constraint concerns the state variable of microorganism respiration. The hypothesis proposed is that the evolution of the respiration rate is an indicator of the microorganism growth necessary for Camembert cheese ripening. This hypothesis was developed on the basis of studies by Couriol et al. (2001) and Adour et al. (2002). The respiration rate should increase up to at least $8.10^{-6} mol.m^{-2}.s^{-1}$ during ripening.

2.3.2. Quality target to be reached

One first important target to be reached for the experts of the factory is the Camembert mass at the end of the ripening process. We established with us a target between $[0.25; 0.27]$ kg. The second important dimension to be taken into account is the microorganism respiration which should ensure good cheese sensory properties. It is fixed at the end of ripening between $[6; 13].10^{-6} mol.m^{-2}.s^{-1}$ for a target of r_{co_2} of $[10, 25] g.m^{-2}.day^{-1}$. The cheese surface temperature is fixed between 281 K and 289K at the end of the ripening process as explained below. The standard time spent in the ripening room is around 12 days before the cheese is wrapped. A first viability kernel was computed with a ripening time of 12 days. The aim was then to evaluate a shorter ripening time. To do this, another viability kernel is calculated for T= 8 days.

2.3.3. The controls

Concerning the controls, the grid is selected upon the sensibility of the sensors and the sensibility of the control systems. The ripening room temperature is chosen from between 281 K and 289 K by increments of 1°K. The relative humidity is chosen from 84% to 98% by increments of 2% (maximum precision of the sensor). The control change (temperature and/or relative humidity) was limited to a frequency of one per 24 h.

2.3.4. The algorithm used to determine the viability kernel

The viability kernel was calculated from the target (end of ripening) to time 0 (beginning of ripening) by means of Algorithm 1. In this algorithm, D_t is the discretised set of the viable state at t, and T is the finite time where the target is reached. The discretised target and the constraints are referred to as C_h and K_h, respectively. The term, $Succ(x)$, represents the successors of x. $Succ(x)$ is the result $(m_{t+1}, T_{s_{t+1}}, r_{co_2 t+1})$ of the Camembert ripening model applied to $x \in K_h$ with position $(m_t, T_{s_t}, r_{co_2 t}, t)$. The viability kernel is built from all of the viable state x at each time interval.

Algorithm 1: **Initialization** $D_T \leftarrow C_h$; **Main loop**; For $t := T - 1$ to 1; $D_t \leftarrow$ $\{x \in K_h | Succ(x) \bigcap D_{t+1} \neq$ ï¿œ$\}$; **Return** $\{D_1, D_2, ..., D_T\}$

2.3.5. High performance computing

The main difficulty in calculating the viability kernel is the dimension of the space to be explored. For example, it is necessary to test 4 150 440 points (controls*states) multiply by 11 days (day 12 = target C) for a ripening time of 12 days. Therefore, 45 654 840 simulations have to be performed with the Camembert ripening model. The calculation time was estimated at 1.5 months on a single computer. As a result, the calculation was distributed in a high performance calculation structure, the MIG-cluster (INRA, Jouy-en-Josas). The viability algorithm was computed with Matlab (The MathWorks, Inc., MA, USA) and then transferred to Octave[1] free software for the calculation distribution. The calculation time was reduced to seven days with the 200 CPU (Central Process Unit) of the MIG-cluster.

2.4. Robustness evaluation of the viable trajectories

The robustness of a viable state does not represent the traditional robustness calculus applied on a controlled system under uncertainties or disturbances. It is a quantification of each viable trajectory as regard to the number of possibilities of control at each time step that leads to viable states. The robustness of a trajectory is simply the sum over time of the relative robustness at each time step.The more are the number of possibilities of control that leads to viable state, the more robust is the viable trajectory. Other possibilities could be considered (such as the min or a discounted sum over the trajectory), see (Alvarez and Martin, to appear).

The robustness of each ripening trajectory $x(.)$ is defined and calculated by

$$Rob(x(.)) := \sum_{t=1}^{T-1} \left(\frac{\sharp Cont_v(x(t))}{\max_{x \in D_t} \sharp Cont_p(x,t)} \right) \tag{6}$$

[1] www.gnu.org/software/octave/

; where $Cont_v(x(t))$ represents the number of viable controls at state $x(t)$ and $Cont_p(x,t)$ the number of possible controls at state $x(t)$.

In our application, some difficulties are encountered to measure and control some variables, like for example the relative humidity of the air Baudrit et al. (2009). If a viable trajectory is calculated as robust, in a given range, it can be demonstrated, using geometric calculus Alvarez and Martin (to appear), that even if the control of the relative humidity is disturbed in this range, the trajectory will keep robust.

2.5. Cheese ripening trials

To test the ripening trajectory found with the viability method, Camembert-type soft mould cheeses were manufactured as described by (Leclercq-Perlat et al., 2004) under aseptic conditions in a sterilised 2-m^3 cheesemaking chamber (figure 1). During, the pilot trial, several indicators were continuously monitored in the ripening chamber : temperature, relative humidity, respiratory activity of the microorganims and cheese mass loss.

Two ripening trials were performed in this study. One trial (SRT) was a standard ripening trial within 12 days in the ripening room at 92% relative humidity and 285 K and the cheeses were wrapped and stored at 277 K. This standard ripening trial is the one typically used in dairy industry. The second trial (TVA) was controlled along the trajectory calculated using the viability approach. The cheeses were ripened for 8 days in the ripening room before being wrapped.

[Figure 1 around here]

The sensory analysis was performed by the sensory analysis company Actilait (Maison du GoÃ»t, Rennes) at day 35 after cheesemaking. This day was choosen as a time reference. The cheeses were evaluated on the basis of 26 indicators on a continuous 10-point scale.The sensory panel also assessed cheeses from a dairy company purchased in a supermarket. The aim was to compare the sensory profile of the experimental cheeses to commercial cheeses as to see if the cheeses ripened under conditions proposed by the viability algorithm could be commercialised. Finally, the data analysis was performed with the Matlab software (The MathWorks, Inc.,MA,USA). A two-way variance analysis (ANOVA) was carried out separately on each attribute according to the following model: attribute = product + repetition + product $\times$ repetition. When significant product differences were observed ($P < 0.05$), product mean intensities were compared using the Tukey-Kramer multiple comparison test.

3. Results

For the viability kernels, we have had to choose explicitly the time duration for reaching the quality target presented 2.3.2. In accordance with experts, we have tested two durations: one with a standard duration encountered in traditional practices, 12 day; an other with a significant reduction of time and as a consequence energy consumption, 8 days. We first present the computed viability kernels for those two different process times (8 and 12 days) in the state space. Surface temperature, as explained below (2.3.1) is not presented in those results. We then describe the results reached within pilot experimentations for the trajectory calculated using the viability approach (TVA). Finally results are compared to a standard ripening trajectory (SRT).

3.1. *Viability kernels*

Two viability kernels were calculated. The discrete viability kernel corresponding to 12 days of ripening is presented in Figure 2. At day 12, the viable states represented correspond to the target C. The kernel is thin at the beginning because the respiration rate is at the 0 level corresponding to the latency phase of microorganisms. At day 1, cheese masses lower than 0.262 kg are not viable and the respiration rate should be at the 0 level. The number of viable respiration rates then reaches a maximum in the middle of the process and decreases at the end.Concerning cheese mass, the viable maximal mass obviously decreases. All the cheese surface temperatures between 281 K and 289 K are viable throughout the process.

[Figure 2 around here]

3.2. *Viable trajectories*

Over all the trajectories registered in the viability kernel, we have kept the most robust ones using the robustness calculus presented section 2.4. In a second step, among those good trajectories we have selected one upon the criteria defined by the cheesemakers from the dairy industry: (1) reduce the initial cheese mass (t0)for a same target at the end of the process, as to reduce the necessary raw material; (2) limit the control variations as to reduce operational costs. One efficient viable trajectory was found for a 0.284 kg cheese at time step t0 and a 8-day ripening period. This trajectory was four days shorter than the standard ripening period. The controls for this trajectory (TVA) are presented in Figure 3b by comparison to the controls for the nominal one typically used in dairy industry (SRT) is presented in Figure 3a.

[Figure 3 around here]

The TVA trajectory differs from the classical one. The relative humidity is constant but 2% higher 94% instead of 92%. The temperature control is modified instead of remaining the same from 285 K at time 1 to 282 K at time 8 with a maximum of 287 K at time 3 and 4. It is in good accordance with the expert knowledge. Indeed, increase the humidity of the air is interesting to limit mass loss without important consequences on the microorganisms. In parallel, previous studies have shown that an increase of the temperature of the air could be interesting for microorganisms growth (Sicard et al., 2011b),(Sicard et al., 2011a).

3.3. Application of the viable ripening trajectory (TVA) on a pilot and comparison to a standard one (SRT)

The TVA trajectory was then applied in a pilot. The results for cheese mass loss evolution, microbiological and physicochemical kinetics were compared to those obtained during standard ripening on this pilot. The sensory quality of the manufactured cheeses was also compared to a commercial one.

3.3.1. Cheese mass loss evolution

Figure 4 shows the mass loss measured during the trial TVA. In the principle of the viability algorithm applied, we have just fixed a set point at the end of the product and constraints for the mass loss all along the ripening process. In this field of constraints and for the set point fixed, the experiment TVA reach well what was attempted with a cheese mass at the end of the ripening within the desired target(0.25 kg-0.27 kg) and a cheese mass between 0.25 kg-0.31 kg during all the process. Compared to the mass loss measured during the standard trial (SRT), the initial value is lower for the TVA than for the SRT with a slope of 0.288kg/jour and 0.251 kg/jour for respectively the SRT and

TVA trajectories. The lower value for the slope of the TVA trajectory can be explained by the difference of relative humidity of the air, 2% higher, limiting the mass transfer. This trajectory is very interesting for experts because with low matter at the input, the mass loss target at the end of the ripening is nevertheless reached due to mass loss reduction. Moreover the time to reach the target, as it could be observed by simulations, is effectively reduced to 8 days. The mass loss is 0.034 kg for the robust ripening and 0.054 kg for the standard ripening. The yield for the viable ripening is about 89% and the one of the standard ripening is about 85%. To conclude, the trajectory selected using the viability algorithm, is in adequation for the mass loss variable, with the criteria fixed in term of constraints and target.

[Figure 4 around here]

3.3.2. Comparison of microbiological and physicochemical kinetics

The repiration rate and microbial activities of viable (TVA) and standard (SRT) ripening processes were also compared. The results are presented in Figure 5 for the respiration rate and microorganisms growth. As projected, the respiration rate began at 0, reached a maximum of over $8.10^{-6} mol.m^{-2}.s^{-1}$ and then slowly decreased until the day the cheese was wrapped. The maximum respiration rate began 1 day earlier in the TVA ripening process than in the standard ripening process but is preserved. Concerning the pH, it increases approximately one day earlier in the TVA ripening process than in the standard one. For the microorganisms growth, differences are limited and kinetics trends are similar for The yeast *K. marxianus*, *G. candidum.* Concerning *B.aurantiacum,* growth occured at the same time for the TVA ripening and for the SRT ripening. However, the level of *B.aurantiacum* was always lower in the case of the viable trajectory.

[Figure 5 around here]

3.3.3. Comparison with commercial camembert cheeses

Cheeses ripened under standard conditions (SRT) and viable conditions (TVA) were assessed by a sensory panel at day 35 and compared to a commercial cheese. The difference between the cheeses was explored with a Tukey-Kramer significance difference test. The results are given in figure 6. The cheese reached with the TVA trajectory was found to be very close to the standard cheese. Only three sensory indicators have revealed significant differences between the cheeses: the core color indicator, the chalky core indicator and the hard texture indicator. It means that for cheese produced under TVA conditions, the quality keep stable with a 4 days reduced ripening time.

[Figure 6 around here]

4. Conclusion

Thanks to the viability theory framework we were able to compute the set of all viable trajectories that satisfy the manufacturing constraint and to reach the quality target for the ripening process. We evaluated a robustness on these trajectories and choose a trajectory with low operational costs from among the more robuste ones. This trajectory has a 8-day ripening time and an initial mass of 0.284 kg, whereas the standard is 12 days and 0.3 kg. This trajectory was validated on a ripening pilot. The microbial equilibrium was preserved so as the cheese sensory properties. We can then conclude that the trajectory built with the viability theory is realistic. The viability method allowed us to effectively propose a pertinent approach of control for the cheese ripening process. It is CPU time consuming. Nevertheless the real value added of this method, by comparison to a control optimal search, is the possibility to describe the whole viable trajectories. As a consequence we are able to calculate the frontier of the viable set and the distance of each trajectory to this frontier. Further studies will be focus on the development of a geometric analysis of the viability kernel for robustness qualification of each trajectories.

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Figure 1: The ripening chamber used for validation trials

Figure 2: The viability kernel calculated for 12 days of ripening. The time, respiration rate and cheese weight viable value for each days are represented.

Figure 3: Relative humidity (dotted line) and temperature (line) to apply to perform a viable robust trajectory. (a) for the standard ripening trajectory (SRT) and (b)for the calculated viable trajectory (TVA).

Figure 4: Cheese mass loss during the experimental trials for the standard ripening trajectory (SRT) and for the viable optimized trajectory(TVA).

Figure 5: Comparative respiration rates and microorganisms growth during an experiment based on the viability calculus (TVA) (line) and a standard experiment (SRT) (dotted line).(a) *pH* (b) *K.marxianus*, (c) *G.candidum*, (d)*P.camemberti*, (*e) B.aurantiacum* (*f) repistation rates.*

Figure 6: Sensory scores for experiment based on the viability calculus (TVA) (line), standard ripened cheeses (SRT) (dotted line) and dairy industry cheeses.

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Table 1: The vector space for the three state variables of the camembert ripening model

	Unit	Min	Max	Steps
Mass	g	250	310	1
Cheese surf temperature	kelvin	281	289	1
Respiration	$g_{CO_2}.m^{-2}.day^{-1}$	0	55	1