Coupling geometric analysis and viability theory for system exploration. Application to a living food system

Salma Mesmoudi ISCPIF, LIP6 salma.mesmoudi@iscpif.fr
Isabelle Alvarez LIP6, IRSTEA isabelle.alvarez@lip6.fr
Sophie Martin IRSTEA sophie.martin@irstea.fr
Romain Reuillon ISCPIF romain.reuillon@iscpif.fr
Mariette Sicard INRA, UMR782 GMPA, mariette.sicard@grignon.inra.fr
Nathalie Perrot INRA, UMR782 GMPA, nathalie.perrot@grignon.inra.fr

October 15, 2014

Abstract

This paper addresses the issue of studying a food complex system in a reverse engineering manner with the aim of identifying the set of all possible actions that makes it reach a quality target with respect to manufacturing constraints. Once the set of actions is identified, several criteria can be considered to identify interesting trajectories and control policies. A viability approach, coupling the viability theory and a geometric approach of robustness, is proposed to study complex dynamical systems. It can be implemented for several types of systems, from linear to non linear or hybrid systems. The proposed framework was adapted to a living food system: a ripening model of Camembert cheese to identify the set of states and actions (capture basin) from which it is possible to reach a predefined quality target. Within the set of viable trajectories, particular trajectories that improve the Camembert cheese ripening process are identified using the proposed approach. The results are applied at a pilot scale and are discussed in this paper.

keywords: model exploration, viability, geometric analysis, food system, optimal control, dynamical system

Nomenclature

t	time	(s)
T and Tfin	time sequence and finite time where the target is	
m	cheese mass	(kg)
ML	mass loss	
r_{o_2}	dioxygen consumption rate	$(mol.m^{-2}.s^{-1})$
r_{co_2}	carbon dioxide consumption rate	$(mol.m^{-2}.s^{-1})$
rh	ripening room relative humidity	(%)
$T°$	ripening room temperature	(Celsius)
RR	respiration rate	$g/m^2/day$
T_s	cheese surface temperature	(Celsius)
DT	disrupted trajectory	
TVA	viable optimized trajectory	
SRT	standard ripening trajectory	
w_{02}	dioxygen molar mass	$(kg.mol^{-1})$
w_{co2}	carbon dioxyde molar mass	$(kg.mol^{-1})$
s	cheese surface	(m^2)
R_T	robustness of trajectory	
mmbr	maximal maximal ball radius	
d	Normalized distance*	

*Normalization of the distance to the boundary of the viability tube: one step is equivalent to 2% in relative humidity or $1°C$ in temperature or 1 g in mass loss or $1m^{-2}.s^{-1}$ in respiration rate.

1 Introduction

The cheese ripening process, such as the one used for Camembert, is considered to be a complex system. Numerous interactions take place over time at different scales, from the microscopic to the macroscopic level. To better understand and eventually enhance Camembert ripening control, numerous studies have been carried out in food sciences. But despite the number of experimental databases collected, process knowledge remains incomplete. Experimental trials are very costly (the necessary time is 41 days per trial). However, models have been developed to help us to more effectively understand such complex processes [29].Cheese processing has been modeled by means of mechanistic models [31], the partial least square method [18], neuronal methods [23],dynamic Bayesian networks [12],genetic algorithms [11], stochastic models [10], finite element methods [14] and the fuzzy symbolic approach [28, 22].Cheese making has been modeled by means of microorganism

kinetics, contamination evolution, substrate consumption, mineral diffusion, sensory property prediction, ripening time prediction and expert knowledge. These models may become a key source to integrate the knowledge from experimental databases. Simulations can be performed with these models to investigate and to better understand food processes.

The aim of this study is to use a viability approach, developed within a complex system approach, to more effectively explore the Camembert ripening process. It relies on an approach coupling the viability theory and a geometric analysis [3]. For basic information on set valued analysis and viability theory, we refer for instance to [5] or [6] and [9]. The viability theory aims at controlling dynamical complex systems with the goal of maintaining them within a given constrained set. Such problems are frequently encountered in ecology or economics where the systems badly deteriorate when they leave some regions of the state space. This theory has been applied to ecological problems such as prey-predator dynamics studied by [16] to determine the necessary conditions to allow prey and predator coexistence. It was also applied to the renewable resource domain, for example, to the viability of trophic interactions in a marine ecosystem by [19] or to the restoration cost of an eutrophic lake by [25] considered as socio-ecological systems. Other applications can also be found in the areas of finance [17], highway traffic fluxes [8] and sociology [15]. This is the first time that viability theory has been applied to a living food system. Moreover the coupling of such an approach with a geometric study, brings original tools to deal with decision help and reverse engineering problems in food science.

Viability theory proposes a framework to calculate the set of all possible states (the capture basin) from which it is theoretically possible to reach a quality target with respect to manufacturing constraints [7]. The ripening process is modeled as a dynamical system. This work is detailed in [34] and [37]. Contrary to optimal control,the viability approach computes the set of all possible states without considering optimization aims at first. The main objective of this method is to keep new and unexpected trajectories that would be disregarded otherwise. The capture basin can be seen as an explicit representation of the process itself.

Once the capture basin has been computed, it is interesting to study its geometric properties. Outside the capture basin, it is not possible to control the ripening process to make an acceptable cheese. So, if a cheese is in a state near the boundary, small perturbations (for example, dysfunction of sensors or actuators, measurements errors, control errors, etc.) can throw the processed cheese out of the capture basin, where it cannot recover. Obviously, trajectories that stay far from the capture basin boundary are safer than trajectories that go near the boundary. The distance to the boundary can

thus be seen as a measure of the robustness of a state to perturbations or errors.

Geometric analysis of the capture basin allows to define the robustness of states and trajectories. Robustness can then be used to qualify particular control sequences. It can also be used explicitly as a selection criterion (as well as control cost and other classical optimization criteria). The global distance analysis can also be used to identify unsafe areas in the process.

This paper presents the geometric study of the capture basin applied to a food living system: the Camembert cheese ripening process, based on the viability study made in [37].

Section 2 introduces the viability framework. Presenting in detail the principles and the algorithm of the geometric method, and the definition of robustness. Section 3 presents the results that were obtained with the Camembert cheese ripening application following by conclusions in section 4.

2 Material and Methods

2.1 Capture basin of the Camembert cheese ripening process.

The domain of viability theory is the control of the dynamical systems [6]. The evolution of the state variable vector $x \in X \subset \mathbb{R}^n$ is described by:

$$\begin{cases} x'(t) & = & f(x(t), u(t)) \\ u(t) & \in & U(x(t)) \end{cases} \tag{1}$$

where $U(x) \subset \mathbb{R}^p$ is the set of admissible controls when the state of the system is x.

Given a constraint set $K \subset X$, the viability theory methods and tools first aim at determining the viability kernel, that is the subset of K gathering all states of the system such that there exists at least one control function that makes it possible to remain in the constraint set indefinitely:

$$Viab(K) = \{x \in K \mid \exists u(.) \mid x(t) \in K \ \forall t \in [0, +\infty[\}.$$

The states belonging to the viability kernel are called viable states.

Given a constraint set K and a target set C, another concept is the capture basin define as all the states for which a control function exists and makes it possible to reach the target set while remaining in the constraint set:

$$Capt(K, C) = \{x \in K \mid \exists u(.),\ \exists T \mid x(T) \in C \text{ and } x(t) \in K \ \forall t \in [0, T]\}.$$

In the case of the Camembert cheese ripening application [37], the state space is 3-dimensional: m, the cheese mass, T_S, the cheese temperature and the microorganism respiration rate, r_{O_2,CO_2}. The cheese mass loss is linked to the evaporation phenomena and the carbon loss through respiration of microorganisms.
The control dynamics for m and T_S are two control variables, the ripening room temperature T_∞ $(Kelvin)$ and the relative humidity rh [20]:

$$\frac{dm}{dt} = s\left\{w_{o_2} \cdot r_{o_2} - w_{co_2} \cdot r_{co_2} - k\left[a_w \cdot p_{sv}(T_s) - rh \cdot p_{sv}(T_\infty)\right]\right\} \qquad (2)$$

$$\frac{dT_S}{dt} = \frac{s}{m \cdot C}\left\{h(T_\infty - T_s) + \varepsilon\sigma(T_\infty^4 - T_s^4) - \lambda k\left[a_w \cdot p_{sv}(T_s) - rh \cdot p_{sv}(T_\infty)\right] + \alpha\frac{r_{o_2} + r_{co_2}}{2}\right\} \qquad (3)$$

a_w is the cheese surface water activity $(dimensionless)$, p_{sv} is the saturation vapor pressure (Pa), k is the average water transfer coefficient $(kg.m^{-2}.Pa^{-1}.s^{-1})$, C is the cheese specific heat $(J.kg^{-1}.K^{-1})$, h is the average convective heat transfer coefficient $(W.m^{-2}.K^{-1})$, ε is the cheese emissivity $(dimensionless)$, σ is the Stefan-Boltzmann value $(W.m^{-2}.K^{-4})$, α is the respiration heat for 1 mol of carbon dioxide release $(J.mol^{-1})$ and λ is the latent vaporization heat of water $(J.kg^{-1})$.

This set of previously established differential equations represents a simple but accurate model to predict cheese mass changes according to available online measurements. The main hypotheses underlying the model are 1) the cheese water activity is constant during ripening, 2) the respiratory activity of the microflora plays a major role by inducing heat production, combined with important water evaporation, 3) the temperature gradient inside the cheese is negligible, and the limiting phenomenon is the convective transfer. The water activity and the specific heat of the cheeses are assessed by offline measurements. The others parameters in the model are obtained from literature.

The controlled dynamics of the microorganism respiration, r_{O_2,CO_2}, are extracted from expert knowledge and experimental results [37].

We fix the following constraints on the state variables: Cheese mass can vary from 250g to 310g, the cheese temperature from 7° C to 17° C and the respiration rate of microorganisms from 0 to 50 $g/m^2/day$.

The sets of admissible controls are ripening room temperature from 8° C to 16° C and relative humidity from 84% to 98%.

The considered manufacturing and quality objectives regarding the sets of admissible controls are described below:

- The final mass of the cheese, must belong must range from $250 to 270$g (the lower limit is fixed by administrative rule),
- The final temperature must remain between $8° - 10°$ C
- The final respiration must remain between 23 and $50 g/m^2/day$
- A specific profile for the gas rate evolution of (CO_2, O_2) is fixed. The respiration rates should begin at level 0 on day 1 (microbial growth latency), reach a maximum between day 3 and day 8 and decrease slowly during the last days of ripening. This temporal constraint is chosen to embed a quality requirement. (see [35][36]).

These four objectives are described by subsets of the state space, also called targets for the viability theory framework [37].

The discretization of the control variables is set upon a balance between (a) the sensitivity of the microorganisms in the cheese to an increment of control variables (known to be significant for an increment of 1° C and 1% of relative humidity [24],[37]) and (b) the precision of the regulation. In ripening rooms, as well as cold chambers [13], spatial variations of humidity and temperature are always observed due to climate control. The resulting gradients deriving from the shape of the room and air regulation devices. Place from an industrial point of view, since it is impossible to place sensors in every point of the ripening chamber, the precision of the regulation is estimated to be around 2%. For the state variables, 1 g in mass loss and $1m^{-2}.s^{-1}$ in respiration rate are fixed. It corresponds to the level of attempted precision described by the experts of the domain for those variables [24], [37].

Given those dynamics, constraints and targets, the viability theory algorithm [32] allows to compute the sequence of T-capture basins, considering constraints depending on time, with backward computation from the target followed by a forward computation from initial conditions (see [37] for more details).

A T-capture basin gathers all states for which there exists a control function that allows to reach the target at time T while remaining in the constraint set between time 0 and T:

$$Capt(T, K, C) = \{x \in K \mid \exists u(.),\ x(T) \in C \text{ and } x(t) \in K\ \forall t \in [0, T]\}.$$

Calculation requires a nearly exhaustive search in the control space at each time step. In this study, the computation is performed by the distributing the effort on a cluster of 200 CPU (Central Processing Unit). A

dimension 5 (plus time) set is produced, the capture basin with the three state variables (cheese mass, respiration rate and cheese temperature) and the two associated control variables (ripening room temperature and relative humidity of the air) [37]. Actually an exhaustive search is made to record all the control values (with respect to the discretization scheme), that lead from one state in $Capt(T-t, K, C)$ to one state in $Capt(T-t+dt, K, C)$, with $t \in [0; T-dt]$.

In the 12 days-capture basin, the traditional Camembert ripening process strategy ($T_s = 12°\text{C}$, $rh = 92\%$) gathers a subset of states (which are all reached using these control values, with exception of initial states, and from which these controls leads to states that are also in the capture basin). All the others states correspond to different possible strategies which eventually lead to the targeted cheese quality.

Once the capture basin is computed, it is possible to perform a geometric analysis in order to identify risky areas. It is also possible to explore it in order to find new control sequences.

2.2 Geometric analysis of a capture basin.

The state space is divided into two areas within the capture bassin: belongs or not to the capture basin. If usual existence assumptions are done on the dynamics, the constraints and the target (compactness, convexity, continuity, for more details see [9]), then the corresponding capture basin is a closed set. We will assume in the following that these assumptions are satisfied and that the capture basin is a closed set. We denote Γ the boundary of the T-capture basin.

In this way, a T-capture basin can be seen as a classification system where the decision boundary is the boundary of the T-capture basin. We thus propose to use technics belonging to the field of classification in order to analyse the capture basin. In particular, the distance to the decision boundary and other geometric concepts can be used to obtain relevant information about a particular state [3].

2.2.1 Geometric robustness

We assume in this section that it is possible to define a meaningful distance in the state space (which dimension is finite), see [4] for more details. In practice, when performing a viability study on a dynamical system, homogenization and rescaling are part of the state variables discretization of in order to run the viability algorithms with efficiency. In the case of the Camembert cheese ripening, the discretization step of state and control variables is de-

termined by domain experts (see [37]), taking into account sensor accuracy. Discretization steps are considered as distance unit on each variable axis. Euclidean distance or sup-norm distance are then defined accordingly. With the sup-norm, the combination of perturbations of the same size on different variables does not change the size of the resulting perturbation.

In this work, we define perturbations one-time instantaneous "shocks" in the state space. Consequently, the occurrence of a perturbation causes a jump in the state space and can be described by a function D that associates state x with state $D(x)$ the reachable state from x after this perturbation [25]. The size of a perturbation is then defined by the size of the jump, the distance $d(D(x) - x)$. A perturbation can be, for instance, an accidental loss of mass, or a sudden change of respiration rate of microorganism (due to external influences such as change of activity or mortality). Former errors of measurement can also be seen as perturbations (once they are discovered).

Considering such perturbations, the distance to the capture basin boundary Γ is a useful indicator of the robustness of a state. If a perturbation occurs when the state of the system is x, as long as its size is smaller than $d(x, \Gamma)$, the new state of the system, $D(x)$, will still belong to the capture basin. Consequently, the distance to the boundary can be seen as a measure of the robustness of a state to perturbations (for instance errors of measurement concerning the localization of the state variable) in the state space. This robustness indicator could be used to assist the process operator when the process is sufficiently monitored: If the state of the system has a low robustness value, it means that it is close to the boundary, so monitoring should be intensified.

We define $p(x)$ as a point of the boundary for which the distance $d(x, \Gamma)$ is reached (for the Euclidean distance it is the orthogonal projection). $p(x)$ gives the direction and size of the smallest perturbation (the sensitive move) at state x that will move the system outside the capture basin.

State robustness can be used to define a robustness of the trajectory followed by the system in the capture basin. Several definitions can be proposed, depending on the manager's perception of risks [30]. The risk of adverse decision can be considered in many different ways. We will consider the most risk-adverse indicator which is the minimum of the distance to the boundary over the trajectory:

Definition 1 ***Min-robustness of a Trajectory.***
Let $T > 0$, $x \in Capt(T, K, C)$ and $x : t \longmapsto x(t)$ be a trajectory starting at x such that $\forall t \in [0; T]$, $x(t) \in Capt(T - t, K, C)$, we note Γ_{T-t} the boundary of $Capt(T - t, K, C)$, then the robustness value of Min-robustness of $x(.)$ is

defined by:

$$r(x(t)) := \min\{d(x(t), \Gamma_{T-t}) | t \in [0; T[\}.$$

With this definition it is possible to assign a robustness value to each cheese ripening trajectory that reaches the target set while satisfying the constraints. Trajectories that stay always far from the T-capture basin boundaries have a higher Min-robustness value, as it can be seen in figure 1. In the following sections we use the term robustness for Min-robustness when dealing with trajectories.

[Figure 1 around here]

When a trajectory has a low Min-robustness value, there are times for which $x(t)$ is close to the boundary of $Capt(T-t, K, C)$ which means that a perturbation (or a measurement error in the state space) with a low intensity may cause a jump outside $Capt(T-t, K, C)$ and the target will no more be reached in the allotted time. Hence, trajectory Min-robustness can be seen as a criterion to choose a particular trajectory among the set of trajectories that reach the target while remaining in the constraint set.

2.2.2 Global quality indicator of a capture basin

Geometric indicators give useful information about the feasibility of controlling the system under the constraints set.

Relative volume. The size of the capture basin compared with the size of the constraint set is a first basic indicator. Since the capture basin V is a subset of the constraint set K, it is easy to compare the size of the corresponding hypervolumes: $\frac{vol(V)}{vol(K)}$.

This indicator is generally used in sensitivity analysis. However, this indicator can gives misleading information about the structure of the problem, even when the capture basin is a simply connected set (which means roughly that it has no holes). because the same volume can enclose rather different shapes. For instance, in a dimension d space, a hypercube with side $2r$ and a hyperrectangle with one side size $2\alpha r$ and all other sides $\frac{2r}{(\alpha)^{\frac{1}{d-1}}}$ with $\alpha >> 1$, share the same volume $(2r)^d$. In the first case, the distance to the boundary of the hypercube reaches its maximum at the center with a value of r. In the second case, the points of the hyperrectangle cannot be farther from the boundary than $\frac{r}{(\alpha)^{\frac{1}{d-1}}}$.

Maximal maximal ball. Maximal balls are open balls included in the capture basin, that are maximal. This means that it is not possible to find a larger open ball centered on the same state that will be totally included in the capture basin. Centers of maximal balls form the skeleton. The maximal maximal ball is the maximal ball with the largest radius among maximal balls. Its center is the farthest point of the capture basin from the decision boundary. The larger the radius of the maximal maximal ball is, the more robust to perturbation the system is: among all the closed sets with the same inner volume, the maximum value of the distance to the boundary is reached for the closed ball.

A relatively large maximal maximal ball suggests that it should be easy to find control sequences that guarantee viability constraints verified over time.

2.3 Distance and projection algorithm

When the decision boundary Γ is described by an analytical formula, or by a set of constraints, it is possible to compute the exact value of $d(x, \Gamma)$ for all points of the input space, with efficient algorithms. When it is not the case, then it is necessary to compute an approximation of the distance. Considering a grid G of N points per axis in dimension n, Γ_N is the set of points that belong to the discretized boundary. When Γ_N is known, efficient algorithms make it possible to compute an approximation of the distance. In practice, Γ_N is often not known and what is computed is the distance to the complementary set (points that are not in the capture basin). An algorithm from [26] has been adapted in [3] to compute on a grid of N points per axis the exact distance to the complementary set. This algorithm is optimal since its complexity is in $O(N^n)$, that is a linear complexity in the number of points of the grid where the distance is computed.

$\bar{V} \neq \emptyset$ is the set of points that are not viable. The distance and projection algorithm *DistanceMapToSet* consists in computing first the distance and a point where the distance is reached along the first axis. The first axis is run in both directions, a function labels points of $\bar{V}$ if any point is discovered, and a counter records the distance to the last encountered point. This procedure is linear in the number of grid points, actually $2N^n$.

Then a second procedure *AdditionalAxis*, is called repeatedly for the $n-1$ remaining axes. Its complexity is also in $O(N^n)$, so the complete algorithm is in $O(n \cdot N^n)$. The basic idea, coming from [21], and detailed in [26] for dimension 2, consists in considering at step k the distance map $g_{k-1}(X)$ computed at step $k-1$ on G (the square distance in the Euclidean case). Building functions F^X specific to the distance are considered at each coor-

dinate x_k along the kth axis, based on the map $g_{k-1}(X)$ computed on the first $k-1$ axes and the distance along axis k. When the difference between two building functions is a monotonic function, then the lower envelope of these functions gives the distance at step k. This monotonicity property is verified in particular in the case of the Euclidean square distance and of the sup-norm distance (we note it D depending on the distance d: $D = d^2$ in the Euclidean case and $D = d$ in the sup-norm case).

In the case of the Euclidean distance, $g_n(X) = d^2(X, p_n(X))$. In the case of the sup-norm, $g_n(X) = d(X, p_n(X))$. (See Appendix for details).

2.4 Theoretical example

We propose here a simple model to illustrate the interest of computing the capture basin and then use the distance to the boundary to select a trajectory robust to perturbations in the state space.

2.4.1 Description of the viability problem

We use in this example a controlled dynamical system, the state space is two-dimensional (x, y) and the control space is one -dimensional u:

$$\begin{array}{lcll} x'(t) & = & x'_0 & x'_0 > 0 \\ y'(t) & = & y'_0 + u(t) & y'_0 > 0 \\ u(t) & \in & [-u_0; +u_0] & u_0 > 0 \end{array} \tag{4}$$

The constraint set is described by:

$$K := \{(x, y) \mid y \in [y_{1,K}; y_{2,K}]\} \tag{5}$$

The target is described by:

$$C := \{(x, y) \mid x = x_1 \text{ and } y \in [y_{1,C}; y_{2,C}]\} \tag{6}$$

The dynamics, the constraint set and the target are displayed in figure 2a.

[Figure 2 around here]

2.4.2 Calculation of the capture basin

In this example, the capture basin is easy to determine as shown in figure 2b.

Outside the capture basin $Capt(K, C)$, it is not possible to reach the target. For states (x, y) which belong to the capture basin (as a topological set), every control value $u \in [-u_0; +u_0]$ applies.

2.4.3 Finding robust evolutions

We consider perturbations in the state space, such as in 2.2.1: a state (x, y) can be translated to $D(x, y) = (x', y')$. The state space is associated with the Euclidean norm. The distance to the boundary of the capture basin as then an index of robustness to these perturbations.

Given an initial point A and knowing the capture basin, we can compute the evolution which is viable and reaches the target, and which also maximizes the integral of the distance to the boundary of the T-capture basins. Such an evolution is displayed in figure 2c.

It is worth noting that the computation of the capture basin should be computed before the determination of this robust evolution.

Indeed, maximizing the distance to the boundary of the constraint set leads to choose a much less robust evolution displayed in figure 2d. This evolution spends a long time on the boundary of the capture basin and the smallest perturbation can provoke a jump of the system state outside the capture basin and the target can no longer be reached.

3 Results: Geometric analysis of the ripening process

The proposed viability algorithm was used to compute the capture basin of the cheese ripening process. We then applied the previously described distance and projection algorithm to this capture basin. The main objectives are to offer a visualization of the ripening process as a set of possible states, to provide global indicators for the capture basin, to study the robustness of theoretical trajectories in order to select the best experimental trajectories.

3.1 Complete capture basin robustness

The complete calculated capture basin is organised in 11th days completed by the capture basin of the 12th day which is the target itself. The dimension of the state space is 3 (see section 2.1).

[Figure 3 around here]

Figure 3 shows the twelve sections (one section per day) of the complete capture basin. The Euclidean square distance (d) to the boundary is used to color the points of each section.

The more the color goes towards red, the more robust the state is (far from boundary), on the contrary, the more the color goes towards blue, the less robust the state is (closer to the boundary).

The shape of the capture basin suggests that the process is rather robust to perturbations in the state space: the daily sections have relatively large maximal maximal balls, with a square radius of the maximal maximal ball of 25 steps which corresponds to a distance d of 5 steps. This is also the theoretical maximum value of distance to the boundary of the constrained domain. This distance can be expressed mathematically, as half the smallest constraint range of the state variables which corresponds to the cheese temperature range, $\frac{10}{2}$ where 10 represents the number of steps between 17 °C and 7°C . The maximum value (5) is reached for an important part of the viable states inside each section from day 4 to day 12, as it can be seen in Figure 3,

the red area of these daily sections covers a significant proportion of the capture basin.

The different sections of the capture basin show the evolution of the possible states of the cheese from one day to the next.

The viability set of day 1 is reduced to a two-dimension set, since the breathing dimension is not active, the microorganisms involved in the process having not germinated yet. All the mass values are viable at the starting day. From the 2nd day until the 6th day (see Figure 3), the microorganisms breathing develops and the viability domain spreads over the constraint set. The percentage of the viable states increases rapidly (see Figure 4) to 89 %.

[Figure 4 around here]

This shows that the exploration domain is correctly defined, since the capture basin is strictly inside the exploration domain but one section occupies almost all the domain.

During the 5th and 6th day, the proportion of viable states is maximal, but the capture basin drifts progressively to higher values of the breathing and higher surface temperature. This can be seen easily in Figure 3. After Day 8 it is no longer possible to reach the target if the microorganisms haven't reached a minimal development. From the 6th day, the viability domain begins converging towards the target. This convergence implies a decrease in the volume of the viable state area, with the fading of high mass values and a slight decrease of the values of the breathing of the microorganisms. The 11th day is very constrained: the percentage of the viable points reaches its lowest level, 17 %, just before the target itself (6 %).

Even in the smaller capture basins, the maximum distance to the boundary remains almost unchanged. This characteristic of the shape of the capture

basin suggests that the ripening process is particularly robust to perturbations.

3.2 Contribution to a reverse engineering approach

The capture basin provides the set of all states from which it is possible to reach the target with respect to the constraints. For each state x in the $(T-t)$-capture basin the viability analysis gives also at least one vector u of control values such that $f(x,u)$ is in the $(T-t+dt)$-capture basin. Actually, in the continuous case, for states that are not on the boundary, all the control values are available. However, some control values may be used only for a very short time. With a constant time step discretization, all the control values may not be available anymore. In our case, with respect to the discretization scheme, we compute all the possible control values that can be associated to a state x. In this enlarged $(T-t)$-capture basin (state space x with possible controls), we used the same definition. In practice, this means that we prefer a state far from the capture basin boundary with control values far from the boundary of available controls.

In accordance with experts, we focus our study on a viability tube computed in less than twelve days, since the analysis of the capture basin showed that it was possible to reach the target in less time. A gain in time of 4 days was considered by the experts as extremely desirable. This is the reason why we have worked on a more constrained set than the complete 12 day tube. We have calculated a new 8 day capture basin and all the associated trajectories.

Inside this capture basin, a trajectory that reaches a weighted compromise between reduced initial mass, reduced operational costs and robustness is selected.

A geometric calculus is applied to this trajectory. On this basis, we show, through simulations and experimental validations on a pilot, that it is possible to predict and quantify the disturbances that could lead to disrupted trajectories of the capture basin, or conversely the perturbations that ensure future states to stay within the boundaries.

added value of the geometric robustness and simulations

The result of the geometric calculus on the 8 day trajectories is presented Table 1.

The geometric robustness of the trajectory is computed with the DistanceMapToSet algorithm with the sup norm in an extended state space. With regards to the identical results for T and T_s, we only use one of the temperatures, labeled

Table 1: The new 8-day trajectory: control values, predicted state values, robustness (distance d to the boundary of the viability tube, dim 4, sup norm), maximal distance to the boundary (mmbr).

	Control value		Ripening state		Distance	
Day	$T\,°C$	rh%	RR	ML	$T_s\,°C$	d(mmbr)
1	12	84	0	284	12	1 (-)
2	13	94	0	280	13	1 (2)
3	14	94	1.7	279	14	3 (4)
4	14	94	6.4	277	14	3 (4)
5	12	94	17.6	276	12	3 (4)
6	12	94	32.1	271	12	3 (4)
7	9	94	44	270	9	3 (4)
8	9	94	43.2	268	-	-

T, for the computation of the distance. This equality can be easily explained by the time to reach T_s knowing the control temperatures. It can be considered as instantaneous as regard to the process reaction time manipulated (1 day). The extended state space is then the 4-dimensions space of the capture basin with normalized state and control variables. The sup norm is used instead of the Euclidean norm, since measurement and control errors can combine but not substitute for one another. A distance (d) of 3, like in day 3, means that one or more variables are 3 steps away from the boundary. Any 2 steps perturbation is not enough to move the cheese state outside. The distance d can be compared to the maximal maximal ball radius of the capture basin(mmbr) for each day. At day 3 for example, a value of 4 means that the thickness of the capture basin at this day (radius of the maximal maximal ball) is 4, that is at least four steps along one variable at least are needed to reach the boundary of the capture basin from its center.

In Table 1 we can see that the 8-days trajectory is very robust, since each day is almost as large as possible (the thickness of the tube is 4).

To assess the validity of our approach, we subsequently test the predicted 8-day trajectory under perturbations.

Since the accuracy of common actuators concerning the control of the relative humidity is rather limited, we decided to apply the disturbance to this variable. Small errors (1%) can induce a great change in the water transfer coefficient and as a consequence in the mass loss [13]. In our model, the discretization step on relative humidity is 2%, to take into account the imprecision of sensors and the difficulty of control.

Since the robustness of the trajectory shown in Table 1 is 3 after the two first days, we apply a perturbation to the relative humidity on day 3 and 4, to be sure to be near the boundary without leaving the viability domain. Instead of the normal value of 94%, we fix a set point of 96% for the relative humidity during these two days. The temperature is unchanged. According to the theoritical results, this disturbance should not bring the trajectory out of the capture basin. Table 2 shows the disrupted trajectory simulated by the model. As expected, the robustness of the disrupted trajectory is now only 2 on day 3 and 4 (it is only two steps from the boundary). The change of state values induced by the change of control values has no other effect on the robustness of the trajectory.

Table 2: 8-days disrupted trajectory: control values, predicted state values, predicted robustness (d), maximal distance to the boundary (mmbr).

	Control value		Ripening state		Distance
Day	$T°$	RH%	RR	ML	d(mmbr)
1	12	84	0	284	1 (1)
2	13	94	2	280	1 (4)
3	14	**96**	7	279	2 (4)
4	14	**96**	19	278	2 (4)
5	12	94	33	277	3 (4)
6	12	94	48	275	3 (4)
7	9	94	41	272	3 (4)

Validation of the simulations on the experimental pilot ripening room

Three trajectories are tested on the INRA's experimental pilot ripening room [36]. The two first are organized in 8 days while the third one is organized in 12 days for a ripening trial close to factory practices. Control and state variables are recorded: temperature ($T°$) and relative humidity (rh%), mass loss (ML), respiration rate (RR). The trajectory corresponding to table (1) control set points, for a ripening in 8 days, is called TVA (viable optimized trajectory). The trajectory corresponding to table (2) control set points, for a ripening in 8 days close to the boundary of the capture basin is called DT (disrupted trajectory). The 12-day trajectory, corresponding to standard controls in a factory for 12 days of ripening is called SRT (standard ripening trajectory, for a control temperature set point of $12\,°C$ and a relative

Table 3: Summary distances (calculated with the algorithm DistanceMapToSet, Euclidean distance in the same 3 dimensional state space as in figure 3 and state values reached during the trial for the standard trajectory SRT

	Control value		Ripening state		Distance	
Day	$T°$	RH%	RR	ML	d	(mmbr)
1	12	92	0	310	1	(1)
2	12	92	3	306	3	(3)
3	12	92	10	302	5	(5)
4	12	92	23	300	5	(5)
5	12	92	43	297	5	(5)
6	12	92	54	294	5	(5)
7	12	92	43	289	5	(5)
8	12	92	37	284	5	(5)
9	12	92	28	280	5	(5)
10	12	92	25	275	4.2	(5)
11	12	92	23	271	-	-

humidity of the air of 92%). This last trajectory is shown to provide a reference for traditional practices. In particular, the robustness of this standard trajectory, as shown in table (3), is almost always at the possible maximum.

The experimental results show that both trajectories reach the designed target. The main impact of the differences of control the slope of mass loss, as it can be seen in figure 6.

The mass loss is smaller for the disturbed trajectory from the 3rd day. Considering the quality of the cheese, it is well known that the range of control of relative humidity, from 96% to 100 especially if this relative humidity is maintained all along the duration of the process, can have a negative impact on the microorganism kinetics and, as a consequence, on the global state of the cheese [24]. In fact, we can see that the respiration rate is modified at the beginning of the ripening process (see figure 7). The profiles of respiration rate for the disturbed and undisturbed trajectories are rather different during days 3 to 5.The respiration rate reached is maximum 1 day earlier by comparison to the SRT and TVA trajectories. Nevertheless, the consequence on the microorganism kinetics for the complete duration of the process, including the time of ripening after wrapping of the cheese (from day 12 to day 45), is very limited, as it can be seen in figure 5.

[Figure 5 around here]

The 8-day trajectory also performs very well in the 8-day capture basin.

Moreover, the proposed approach also helps us to know to which extent this new way of control is robust, and what is the acceptable uncertainty with regards to the boundary of the capture basin. This is very encouraging for further developments of control strategies at 8-day ripening rather than the standard of 12-day process.

[Figure 6 around here]

[Figure 7 around here]

4 Conclusions

Thanks to the viability theory framework applied to the ripening process, we are able to compute the set of initial points included in a given constrained set from which starts at least one evolution (1) satisfying the manufacturing constraints and (2) reaching the quality target at the allotted time.
We evaluate the robustness of these trajectories by using recent original mathematical developments, finally choosing a trajectory with low operational costs among the more robust ones. This trajectory has a 8-day ripening time and an initial mass of 0.284 kg, whereas the standard one was ripened in 12 days from an initial mass of 0.3 kg. Moreover, we show that a coupling between the viability method and an adapted geometrical robustness computation leads to interesting information for reverse engineering purposes. We show that it is possible to predict and quantify the disturbances that can lead to disrupted trajectories that go outside the viable set and conversely that can ensures the state to stay in the viable set. This allows us to effectively control the cheese ripening process in an optimized way. The method developed in this work can be applied to other processes for which a dynamical model is available, in order to explore its state space and propose new ways of controlling it.

Robustness is crucial for the dairy industry to avoid process drift. Other measures of robustness based on the capture basin boundary should be developed to identify the areas where the process is more sensitive to perturbation and, consequently, where it should be more carefully monitored.

In future works, we will investigate in more detail the enlarged state space with viable controls, and the links with the time of exit from the capture basin (since some control values can apply for very short time if a state is near the capture basin boundary).

Further studies will be focus on the exploitation of this approach to consider a multi-objective optimization problem related to the modeling of a food industrial process, that is the Camembert cheese ripening process. One aim is to construct the Pareto boundary [27] of the optimal strategies, i.e. the optimal paths, among each possible viable path to the target.

Acknowledgment

We thank Leclercq-Perlat M.N., Lecornue, F., Guillemin H., Savy, M., Picque D. for the experiments, Bourgine, P. for the ideas, Tonda, A. for the rephrasing and English, the French ANR for the grant for the INCALIN project and the funding from the European Community's Seventh Framework Programme (FP7/2007-2013) under the grant agreement n°FP7-222 654-DREAM.

Appendix: Geometric Algorithms

Algorithms are adapted from [26]. We consider a grid G of N points per axis in dimension d. $\bar{V}$ is the set of points that are not viable. Since the boundary of V is not known explicitly, this kind of algorithm is appropriate (when the boundary is known, other algorithm are more suitable). We note $X = (x_1, .x_k, .x_n)$ and $X_k = (x_1, .x_{k-1}, x_{k+1}, .x_n) \in [0, N-1]^{n-1}$, we note $g_k^{X_k}(x_k) = g_k(X)$ the distance map (we note it D depending on the distance d: $D = d^2$ in the Euclidean case and $D = d$ in the sup-norm case) and $p_k^{X_k}(x_k) = p_k(X)$ the point where it is reached. If we note E_{k-1} the subspace spanned by the $(k-1)$ first axis, we have:

$$\forall X \in G,\ \bar{V} \cap (X + E_{k-1}) \neq \emptyset \Rightarrow \begin{cases} g_{k-1}(X) = D(X, \bar{V} \cap (X + E_{k-1})) \\ \exists Y = p_{k-1}(X) \in \bar{V},\ D(X,Y) = g_{k-1}(X) \end{cases} \tag{7}$$

The definition of the building functions depends on D and is such that:

$$F^X(i) = F_{x_k}^{X_k}(i) := D\left((x_1, .x_{k-1}, i, x_{k+1}, .x_n), p_{k-1}(X)\right) \tag{8}$$

The minimum of a building function $F_{x_k}^{X_k}(i)$ is reached for $i = x_k$.

In the case of the square Euclidean distance, the definition of the building functions from equation (8) is given by the Pythagorean theorem and the coordinate of their intersection on the kth axis (noted u_k) on the grid is

defined by:

$$\left\{F^X(i) = F^{X_k}_{x_k}(i) := g_{k-1}(X) + (i - x_k)^2\right\} (0 \leq x_k < N)$$
$$\text{intersect}(F^{X_k}_{x_k}, F^{X_k}_{y_k}) = \text{truncate}(x_k^2 - y_k^2 + g_{k-1}(X) - g_{k-1}(Y)) \div 2(x_k - y_k) \tag{9}$$

In the case of the sup norm, the building functions (8) are truncated V-shaped function defined by:

$$F^X(i) = F^{X_k}_{x_k}(i) := \max(g_{k-1}(X), |i - x_k|),\ (0 \leq x_k < N)$$

The intersection of truncated V-shaped functions F^X and F^Y used by the algorithm is given by the following formula (with $x_k \leq y_k$), contrary to [26]:

$$\text{intersect}(F^X, F^Y) = \begin{cases} \max((x_k + g_{k-1}(Y)), ((x_k + y_k) \div 2)) \\ \qquad\qquad\qquad\qquad \text{if } g_{k-1}(X) \leq g_{k-1}(Y) \\ \min((y_k - g_{k-1}(X)), ((x_k + y_k) \div 2)) \text{ otherwise} \end{cases} \tag{10}$$

The algorithm can be used for other distances, as long as the difference of building functions verifies the monotonicity property (see [21]).

We now consider the kth axis u_k, $2 \leq k \leq n$. $F^{X_k}_l(x_k)$, with $0 \leq l < N$, gives the distance (or square distance in the Euclidean case) between $X = (x_1, .x_k, .x_n)$ and $p^{X_k}_{k-1}(l)$. We have $F^{X_k}_l(x_k) = F^{(X+(l-x_k)u_k)}(x_k)$. If $\bar{V} \cap (X + E_k) \neq \emptyset$, then there is at least one l, $0 \leq l < N$ such that $g^{X_k}_{k-1}(l)$ and $p^{X_k}_{k-1}(l)$ are defined. Then we have:

$$\begin{cases} g_k(X) = \min\left\{F^{(X+(l-x_k)u_k)}(x_k),\ 0 \leq l < N\right\} \\ p_k(X) = p_{k-1}(X + (m - x_k)u_k) \\ \qquad\qquad \text{with } m = \text{argmin}\left\{F^{(X+(l-x_k)u_k)}(x_k),\ 0 \leq l < N\right\} \end{cases} \tag{11}$$

But the computation of all these values of F is suboptimal, so the algorithm considers the set of building functions $F^{X_k}_l$, and computes the lower envelope of this set, using the intersection between two building functions F to switch from one function to another.

For all points $X_k \in [0, N-1]^{n-1}$, the procedure AdditionalAxis recruits building functions along axis k. A candidate function $f = F^{X_k}_i$ is recruited if there is no other recruited function or if it intersects the previous recruited function $F^{X_k}_j$ with $j < i$ on the grid. From the definition of the building functions, we then have:

$$\text{Let } 0 \leq j < i < N \text{ and } w = \max(0; 1 + \text{intersect}(F^{X_k}_j, F^{X_k}_i)),$$
$$(w < N) \Rightarrow \forall l,\ w \leq l < N,\ F^{X_k}_i(l) < F^{X_k}_j(l) \tag{12}$$

The candidate function is then recruited after $F_j^{X_k}$ from the coordinate w if $w < N$. If there is no previous recruited function it is recruited from $w = 0$.

Let us suppose that there are already two building functions $F_{x_1}^{X_k}$ and $F_{x_2}^{X_k}$ with $x_1 < x_2 < i$. $F_{x_2}^{X_k}$ was recruited from $w(x_2)$ with:

$$0 \leq w(x_2) = 1 + \text{intersect}(F_{x_1}^{X_k}, F_{x_2}^{X_k}) < N$$

. Then if $f(w(x_2)) = F_i^{X_k}(w(x_2)) < F_{x_2}^{X_k}(w(x_2))$, the monotonicity property insures that for all $l \geq w(x_2)$, then $f(l) < F_{x_2}^{X_k}(l)$ (This can be easily verified from equations (9) and (10) respectively). So the candidate f replaces the last function $F_{x_2}^{X_k}$. This is done until all previously recruited functions are replaced or alternatively until the last recruited function $F_j^{X_k}$, recruited from w_j with $j < i$ is such that $F_j^{X_k}(w_j) < f(w_j)$. In that case f is recruited with w as in (12).

Let $(F_{x_i}^{X_k})_{i \in I}$ be the sequence of building functions recruited along axis k from $(w_i)_{i \in I}$ respectively. The functions are labeled such that if $i < j$ then $x_i < x_j$. The sequence of recruitment points (w_i) defines the lower envelope of the recruited building function. We then have:

$$\forall x_k,\ 0 \leq x_k < N,\ \text{let } m = \max\{i \in I,\ w_i \leq x_k\} \text{ then}$$
$$\begin{cases} g_k(X) = g_k^{X_k}(x_k) = F_{x_m}^{X_k}(x_k) \\ p_k(X) = p_{k-1}^{X_k}(x_m) \end{cases} \tag{13}$$

In the case of the Euclidean distance, $g_n(X) = d^2(X, p_n(X))$. In the case of the sup-norm, $g_n(X) = d(X, p_n(X))$. $p_n(X)$ is a point where the distance is reached (the orthogonal projection in the Euclidean case).

References

[1] Aggarwal, A., Klowe, M., Moran, S., Shor, P., Wilber, R. (1987)Geometric applications of a matrix-searching algorithm. Algorithmica. 2. 195–208.

[2] Alvarez, I. (2004) Explaining the result of a Decision Tree to the End-User. Proc. of the 16th European Conference on Artificial Intelligence. 411–415. IOS Press.

[3] Alvarez, I., Martin. S., Mesmoudi, S. (2010) Describing the Result of a Classifier to the End-User: Geometric-based Sensitivity. Proc. of the 19th European Conference on Artificial Intelligence. 835–840. IOS Press. http://dx.doi.org/10.3233/978-1-60750-606-5-835.

[4] Alvarez, I. and Martin. S. (2011) Geometric robustness of viability kernels and resilience basins. In Viability and resilience of complex systems: concepts, methods and case studies from ecology and society. Springer. 193–218.

[5] Aubin J.-P., Frankowska H. (1990) Set-Valued Analysis. Birkhaüser, Boston, Basel.

[6] Aubin, J.-P. (1991) Viability theory. Birkhaüser, Boston, Basel.

[7] Aubin J.-P. (2001) Viability kernels and capture basins of sets under differential inclusions. SIAM Journal of Control and Optimization. 40, 853-881

[8] Aubin, J. P., Bayen, A. M., Saint-Pierre, P. (2005) A viability approach to hamilton-jacobi equations:application to concave highway traffic flux functions. 44th IEEE Conference on Decision and Control & European Control Conference, Vols 1-8, 3519–3524.

[9] J.-P. Aubin, A. Bayen and P. Saint-Pierre, *Viability Theory. New Directions.* (Springer, 2011).

[10] Aziza, F., Mettler, E., Daudin, J. J., Sanaa, M. (2006) Stochastic, compartmental, and dynamic modeling of cross-contamination during mechanical smearing of cheeses. Risk Analysis: 26 (3) 731–745.

[11] Barriere, O., Lutton, E., Baudrit, C., Sicard, M., Pinaud, B., Perrot, N. (2008) Modeling human expertise on a cheese ripening industrial process using gp. Parallel Problem Solving From Nature - Ppsn X, Proceedings 5199, 859–868.

[12] Baudrit, C., Wuillemin, P. H., Sicard, M., Perrot, N. (2008) A dynamic bayesian network to represent a ripening process of a soft mould cheese. Knowledge-Based Intelligent Information And Engineering Systems, Pt 2, Proceedings 5178, 265–272.

[13] Baudrit, C., Hélias, A., and Perrot, N. (2009) A Joint treatment of imprecision and variability in food engineering: Application to cheese mass loss during ripening. Journal of Food Engineering. 93. 284–292.

[14] Bona, E., da Silva, R. S. S. F., Borsato, D., Silva, L. H. M., Fidelis, D. A. D. (2007) Multicomponent diffusion modeling and simulation in prato cheese salting using brine at rest: The finite element method approach. Journal of Food Engineering 79(3), 771–778.

[15] Bonneuil, N. (2000) Viability in dynamic social networks. Journal Of Mathematical Sociology 24 (3),175–192.

[16] Bonneuil, N., Mullers, K. (1997) Viable populations in a prey-predator system. Journal Of Mathematical Biology 35 (3), 261–293.

[17] Bonneuil, N. et Saint-Pierre, P. (2004) The hybrid guaranteed capture basin algorithm in economics.

[18] Cabezas, L., Gonzalez-Vinas, M., Ballesteros, C., Martin-Alvarez, P. J. (2006) Application of Partial Least Squares regression to predict sensory attributes of artisanal and industrial Manchego cheeses. *European Food Research and Technology*,222(3-4), 223–228.

[19] Chapel, L., Defluant, G., Martin, S., Mullon, C. (2008) Defining yield policies in a viability approach. Ecological Modelling 212 (1-2), 10–15.

[20] Hélias, A., Mirade, P. S., Corrieu, G. (2007)Modeling of Camembert-type cheese mass loss in a ripening chamber: Main biological and physical phenomena. *Journal of Dairy Science*, 90, 5324-5333.

[21] Hirata, T. (1996)A Unified Linear-Time Algorithm for Computing Distance Maps. Information Processing Letters. 58 (3). 129–133.

[22] Ioannou, I., Perrot, N., Mauris, G., Trystram, G. (2003) Experimental analysis of sensory measurement imperfection impact for a cheese ripening fuzzy model. Fuzzy Sets and Systems - Ifsa 2003, Proceedings 2715, 595–602.

[23] Jimenez-Marquez, S. A., Lacroix, C., Thibault, J. (2003) Impact of modeling parameters on the prediction of cheese moisture using neural networks. Computers & Chemical Engineering 27 (5), 631–646.

[24] Leclercq-Perlat, M. N., Buono, F., Lambert, D., Latrille, E., Spinnler, H. E., Corrieu, G. (2004) Controlled production of Camembert-type cheeses. part i: Microbiological and physicochemical evolutions. Journal of Dairy Research 71 (3). 346–354.

[25] Martin, S., Dec. (2004) The cost of restoration as a way of defining resilience: a viability approach applied to a model of lake eutrophication. Ecology And Society 9 (2), 8.

[26] Meijster, A. and Roerdink, J., Hesselink, W.H. (2000) A General Algorithm for Computing Distance Transforms in Linear Time. Morphology and Its Applications to Image and Signal Processing. 331-340.

[27] Pareto. V. (1912) Manuel d'Ã©conomie politique. Bull. Amer.

[28] Perrot, N., Ioannou, I., Allais, I., Curt, C., Hossenlopp, J., Trystram, G. (2006). Fuzzy concepts applied to food product quality control: a review. Fuzzy Sets and Systems, 157, 1145-1154.

[29] Perrot, N., Baudrit, C., Trelea, I.C., Trystram, G., Bourgine, P. (2011)Modelling and analysis of complex food systems: state of the art and new trends. *Trends in Food Science and Technology*, 22(6), 304-314.

[30] Plous, S. (1993) The Psychology of Judgment and Decision Making. McGraw-Hill.

[31] Riahi, M. H., Trelea, I. C., Picque, D., Leclercq-Perlat, M. N., Helias, A., Corrieu, G. (2007) Model describing Debaryomyces hansenii growth and substrate consumption during a smear soft cheese deacidification and ripening. *Journal of Dairy Science*, 90 (5), 2525-2537.

[32] P. Saint-Pierre, Approximation of the viability kernel, *Appl. Math. Opt.* **29** (1994) 187–209.

[33] Sicard,M., Perrot,N., Baudrit, C., Reuillon, R., Bourgine,P., Alvarez, I.,Martin, S. (2009) The viability theory to control complex food processes. European Conference on Complex Systems.

[34] Sicard,M. (2010) Methods, concepts and tools of system complex to control food processes. Camembert cheese ripening application. AgroParisTech PhD dissertation. http://tel.archives-ouvertes.fr/tel-00450100. (In French).

[35] Sicard, M., Baudrit, C., Leclerc-Perlat, M.N., Wuillemin, P.H., Perrot, N. (2011) Expert knowledge integration to model complex food processes. Application on the camembert cheese ripening process. *Expert Systems with Applications*, 38(9), 11804-11812.

[36] Sicard, M., Perrot, N., Leclerc-Perlat, M.N., Baudrit, C., Corrieu, G. (2011) Towards Integration of Experts Skills and Instrumental Data to control food processes. Application to Camembert-type cheese ripening. *Journal of Dairy Science*, 94(1), 1-13.

[37] Sicard, M., Perrot, N., Reuillon, R., Mesmoudi, S., Alvarez, I., Martin, S. (2011) A viability approach to control food processes: Application to a Camembert cheese ripening process. *Food Control*, 23, 312-319.

List of figures:

Figure 1: Geometric robustness of trajectories in the capture basin. The robustness of $x(t)$ is low whereas the robustness of $y(t)$ is high.

Figure 2: Construction of a capture basin. , (a) *dynamics (4), constraint set (5) and target set (6)*, (b) *Capture basin*, (c) *From initial point A, the blue evolution, viable and which reaches the target, also maximises the integral of the distance to the boundary of the capture basin. The green curve describes the skeleton of the capture basin*, (d) *From initial point A, the red evolution, viable and which reaches the target, also maximises the integral of the distance to the boundary of the constraint set. The black dotted curve describes the skeleton of the constraint set.*

Figure 3: Distance square map of the capture basin (12 sections corresponding to the different days of the process). Viable states that are near the boundary are not shown (the yellow bounding box shows the maximal extent of viable states). Axes x(red), y(green) and z(blue) correspond to the mass(g), temperature (°C) and the respiration rate of microorganisms ($g/m^2/day$) respectively. The black box delimits the target: final mass between 251 and 276g, final temperature between 8 and 10°C, respiration rate between 34 and $55g/m^2/day$.

Figure 4: Ratio of the number of viable states.

Figure 5: Evolution of the microorganisms during the cheese ripening for TVA (viable optimized trajectory) and DT (disrupted trajectory), (a) *K. marxianus*, (b) *G. candidum*, (c) *B. aurantiacum.*

Figure 6: Comparative profile of the mass loss reached for three experiments led on the experimental pilot TVA (viable optimized trajectory), DT (disrupted trajectory) and SRT (standard ripening trajectory).

Figure 7: comparative profile of the respiration rate variation reached for the three experiments TVA, DT and SRT led on the pilot.