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Defining yield policies in a viability approach

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ARTICLE INFO

Article history:
Published on line 26 November 2007

Keywords:
Viability theory
Marine ecosystem
Fisheries management
Support vector machines

ABSTRACT

Mullon et al. [Mullon, C., Curry, P., Shannon, L., 2004. Viability model of trophic interactions in marine ecosystems. Nat. Resour. Model. 17 (1), 27–58] proposed a dynamical model of biomass evolution in the Southern Benguela ecosystem, including five different groups (detritus, phytoplankton, zooplankton, pelagic fish and demersal fish). They studied this model in a viability perspective, trying to assess, for a given constant yield, whether each species biomass remains inside a given interval, taking into account the uncertainty on the interaction coefficients. Instead of studying the healthy states of this marine ecosystem with a constant yield, we focus here on the yield policies which keep the system viable. Using the mathematical concept of viability kernel, we examine how yield management might guarantee viable fisheries. One of the main practical difficulties up to now with the viability theory was the lack of methods to solve the problem in large dimensions. In this paper, we use a new method based on SVMs, which gives this theory a larger practical potential. Solving the viability problem provides all yield policies (if any) which guarantee a perennial system. We illustrate our main findings with numerical simulations.

1. Introduction

The viability theory (Aubin, 1991) aims at controlling dynamical systems with the goal to maintain them inside a given set of admissible states, called the viability constraint set. Such problems are frequent in ecology or economics, when systems die or badly deteriorate if they leave some regions of the state space. For instance Bene et al. (2001) studied the management of a renewable resource as a viability problem. They pointed out irreversible overexploitation related to the resource extinction. Bonneuil (2003) studied the conditions the prey-predator dynamics must satisfy to avoid extinction of one or the other species as a viability problem. Cury et al. (2005) consider viability theory to advise fisheries.

Mullon et al. (2004) proposed a dynamical model of biomass evolution of the Southern Benguela ecosystem, involving five different groups (detritus, phytoplankton, zooplankton, pelagic fish and demersal fish). They studied this model in a viability perspective (Aubin, 1991), trying to assess, for a given constant yield, whether each species biomass remains inside a given interval, taking into account the uncertainty on the interaction coefficients. The aim was to identify constant yield values that allow persistence of the ecosystem. We extend the problem and we focus here on the yield policies which keep the system viable, instead of considering a constant yield.

Using the mathematical concept of *viability kernel*, we examine how yield management might guarantee viable fisheries. The *viability kernel* designates the set of all viable states, i.e. for which there exists a control policy maintaining them within the set of constraints. Outside the viability kernel, there is no evolution which prevents the system from collapsing. Aubin (1991) proved the viability theorems which enable to determine the viability kernel, without considering the combi-

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0304-3800/$ – see front matter
doi:10.1016/j.ecolmodel.2007.10.007

natorial exploration of control actions series. These theorems also provide the control functions that maintain viability.

This general approach shows several interesting specific aspects:

- It can take into account the uncertainties on the parameters which are generally high in ecosystem modelling. Here, we manage the uncertainties like in Mullon et al. (2004).
- The viability kernel can define a variety of different policies, which respect the viability constraints. Therefore, it offers more possibilities for negotiations and discussions among the concerned stakeholders than techniques which propose a single optimal policy.

The main limitation of the viability approach is its computational complexity. The existing algorithm for viability kernel approximation (Saint-Pierre, 1994) supposes an exhaustive search in the control space at each time step. This makes the method impossible to use when the control space is of a 51 dimensions like in our problem. Mullon et al. (2004) solved this problem with a method which is only adapted to linear equations of evolution. Here, we use a new method, based on support vector machines, which can be applied to non-linear models as well (Deffuant et al., 2007).

We present the viability model of the Southern Benguela ecosystem and we recall the main concepts of the viability theory. Then, we describe our main numerical results. We show the shape of the found viability kernel, and the corresponding possible yield policies. Finally, we discuss the results and draw some perspectives.

2. The viability model of the Southern Benguela ecosystem

Following a classical approach (Walters and Pauly, 1997), we suppose that the variation of the biomass of species i due to its predation by other species j depends linearly on the recipient and donor biomasses (B_j and B_i), with respective coefficients r_{ji} and d_{ji}. The biomass lost by species i due to the predation by the other species is expressed by Eq. (1):

$$\frac{dB_i(i\rightarrow)}{dt} = -\sum_j (r_{ji}B_j + d_{ji}B_i). \quad (1)$$

The variation of the donor biomass B_i due to this interaction takes into account the assimilation of the biomass of other species j, multiplied by a growth efficiency coefficient (denoted below by g_i). Therefore, the biomass gained by species i, because of its consumption of other species, is expressed by:

$$\frac{dB_i(i\leftarrow)}{dt} = g_i \sum_j (r_{ij}B_i + d_{ij}B_j). \quad (2)$$

For the detritus, the variation of the biomass follows the same principle, but it also integrates the non-assimilated biomass of the other species, except phytoplankton, which is added to the detritus biomass B_1 (multiplied by its growth efficiency g_1):

$$\frac{dB_1(\text{non-assimilated})}{dt} = \sum_{j>2}\sum_k g_1(1-g_l)(r_{lk}B_l + d_{kl}B_k). \quad (3)$$

The model of the Southern Benguela ecosystem considers trophic interactions (predation, consumption and catch) among five components: detritus (i = 1), phytoplankton (i = 2), zooplankton (i = 3), pelagic fish (i = 4), demersal fish (i = 5). In total, the biomass evolution can be written as follows:

$$\frac{dB_1}{dt} = \frac{dB_1(1\leftarrow)}{dt} - \frac{dB_1(1\rightarrow)}{dt} + \frac{dB_1(\text{non-assimilated})}{dt} - Y_1,$$
$$\frac{dB_i}{dt} = \frac{dB_i(i\leftarrow)}{dt} - \frac{dB_i(i\rightarrow)}{dt} - Y_i, \quad (4)$$

where g_i is the growth efficiency of species i, Y_i is the yield of species i. Fig. 1 shows the structure of the ecosystem.

Mullon et al. (2004) take into account the uncertainty on parameters r_{ij} and d_{ij}, which is expressed by:

$$r_{ij} \in \left[\bar{r}_{ij} - \delta r_{ij}, \bar{r}_{ij} + \delta r_{ij}\right], \qquad d_{ij} \in \left[\bar{d}_{ij} - \delta d_{ij}, \bar{d}_{ij} + \delta d_{ij}\right]. \quad (5)$$

They consider this model in a viability perspective, in order to study the persistence of the ecosystem and to define the impact of the fisheries. Given a constant yield, they define scenarios which result is a "healthy" system.

Extending the work of (Mullon et al., 2004), we incorporate the fisheries in this study as a control variable of the system, in order to find the yield policies which allow keeping the system viable. To guarantee a perennial system, the viability

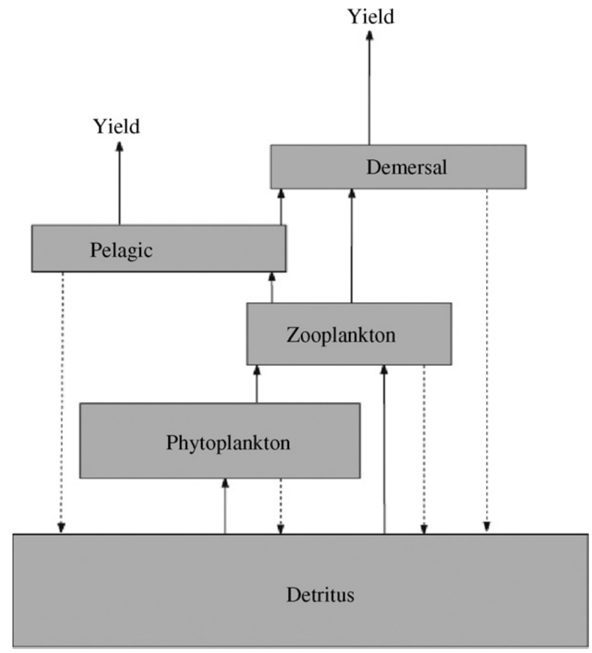

Fig. 1 – Components and structure of the Southern Benguela ecosystem. Arrows represent the flux between compartments (from Mullon et al., 2004).

constraints are defined by:

$$0 \le m_i \le B_i \le M_i, \qquad 0 \le y_{min} \le Y_i \le y_{max},$$
$$Y'_i \in [-\delta y, +\delta y], \quad i = 4, 5, \tag{6}$$

where m_i is the minimum level for the resource, M_i the maximal biomass that can be contained in the ecosystem, y_{min} the minimum level for yield for demersal and pelagic fish, and y_{max} the maximum level. The parameter δy limits the evolution of the fisheries between two time steps. We suppose that the levels of yields of pelagic fish and demersal fish are the same. These constraints, which attain critical values of a "healthy" system allow one to link yield objectives with the principle of ecosystem persistence.

3. The viability analysis control problem and viability kernel approximation

In the viability problem, the controls are the yields on pelagic fish (Y_4), demersal fish (Y_5), and the uncertainty on coefficients r_{ij} and d_{ij}. This means that for any state of the system located in the viability kernel, there exist values of these parameters for which the system remains in the viability kernel at the next time step. Adding the constraints on the derivatives of Y_4 and Y_5 implies to add two dimensions to the state space, which would then be 7. This reaches the current computational limits, therefore, we supposed that $Y_4 = Y_5 = Y$. This hypothesis is of course not realistic, but we thought it would nevertheless be an interesting first step.

The viability control problem is to determine a control function:

$$t \to r_{ij}(t),\ d_{ij}(t), Y'_4(t), Y'_5(t) \quad \text{with } i, j = 1, 2, 3, 4, 5 \tag{7}$$

which enables to keep the viability constraints (6) satisfied indefinitely. Solving this problem requires to determine the viability kernel, which is the set of states for which such a control function exists.

Saint-Pierre (1994) proposed an algorithm to approximate the viability kernel from the problem defined on a grid but the result is a set of points that is viable and it requires an exhaustive search in the control space, which is not possible in our case because the control is in the dimension 51.

To approximate the viability kernel of the Southern Benguela ecosystem, we use a new algorithm (Deffuant et al., 2007) (see Appendix 1) which is built on previous work from Saint-Pierre (1994), using a discrete approximation of the viability constraint set K by a grid. Its main characteristic is to use an explicit analytical expression of the viability kernel approximation, in order to make it possible to use standard optimization methods to compute the control. This analytical expression is provided by a classification procedure, the support vector machines (SVMs) (Vapnik, 1998; Cristianini and Shawe-Taylor, 2000). This algorithm is interesting in the case we study, because the analytical expression of the viability kernel allows to use optimization techniques in order to find the best evolution in high dimensional control spaces.

Table 1 – Estimation of the minimal and maximal biomasses (B_i) for the five species

Compartment	m_i (tonnes/km^2)	M_i (tonnes/km^2)
Detritus	100	2000
Phytoplankton	30	400
Zooplankton	20	200
Pelagic fish	5	60
Demersal fish	5	30

4. Numerical simulations

The donor and recipient control coefficients are derived from a mass-balanced Ecopath model for the ecosystem (Shannon et al., 2003). We use the evaluation of the parameters provided in (Mullon et al., 2004). Table 1 gives the values of the viability constraint set and we put $y_{min} = 0$ tonnes/km^2 (no catches at all), $y_{max} = 5$ tonnes/km^2 (the minimal level of the biomass of pelagic and demersal fish, corresponding on the maximum constant value tested by Mullon et al. (2004)), $\delta y = 0.5$ (which represents a variation of 10% of the maximal yield). The yield for others species has been set to 0, except for detritus ($Y_1 = -1000$ tonnes/km^2, which correspond of an import of detritus).

The following figures present some results for given values of biomasses of each species. The boundaries of the axes are the constraints defined on the species represented. The approximation of the viability kernel is represented in grey. Inside the viability kernel, there is at least one viable path which allows keeping a healthy system and outside, there is no evolution which prevents the system from collapsing. We focus here on values of detritus biomass = 2000 tonnes/km^2 because this ensures the existence of a viability kernel for almost all the values of the others compartments. For a level of detritus biomass = 1600 tonnes/km^2, given values of zooplankton and phytoplankton are necessary to guarantee a viable path. For lower detritus biomass, there is no viable path: a threshold of detritus biomass is necessary for ensuring a perennial system.

In the algorithm used to approximate the viability kernel (Deffuant et al., 2007), we used a grid with 6 points per dimension (46,000 points in total) and 1642 support vectors are necessary to define the boundary of the kernel.

We focus on the effects of fisheries on demersal and pelagic fish.

4.1. *Effects of fisheries on demersal fish*

Fig. 2 presents a 2D slices of the viability kernel where detritus biomass = 2000 tonnes/km^2, phytoplankton = 100 tonnes/km^2, zooplankton = 90 tonnes/km^2 and for different values of pelagic fish biomass. Horizontal axis represents demersal fish and vertical one the fisheries.

The levels of pelagic fish, demersal fish and yield have an influence on the boundary of the viability kernel:

- For low values of pelagic fish biomass, the demersal fish biomass must not be too high and consequently intensive

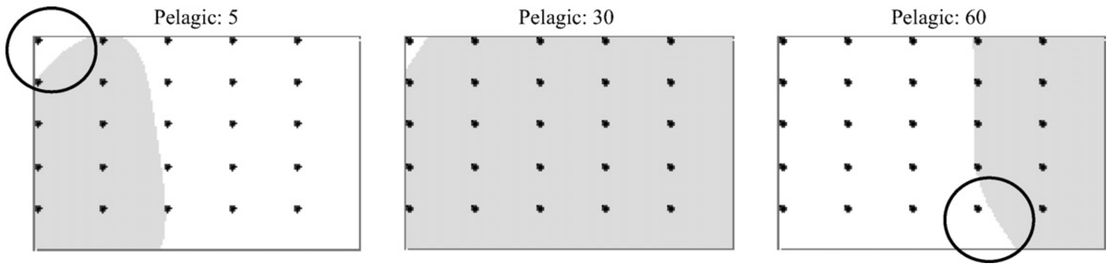

Fig. 2 – Approximation of viability kernel. The horizontal axis represents demersal fish, vertical axis fisheries, detritus = 2000 tonnes/km^2, zooplankton = 90 tonnes/km^2 and phytoplankton = 100 tonnes/km^2.

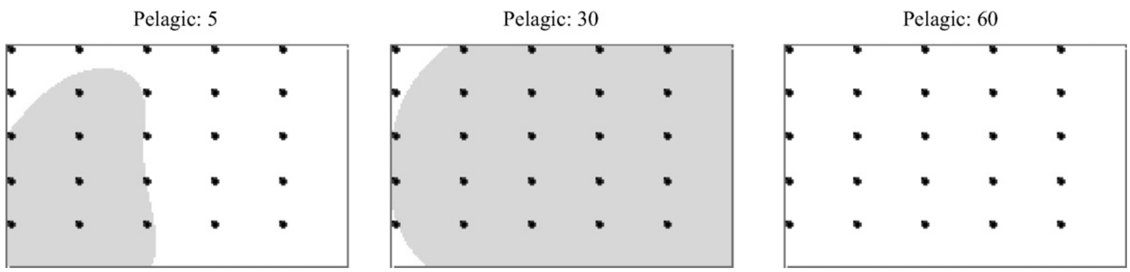

Fig. 3 – Approximation of viability kernel. The horizontal axis represents demersal fish, vertical axis fisheries, detritus = 2000 tonnes/km^2, zooplankton = 130 tonnes/km^2 and phytoplankton = 400 tonnes/km^2.

fishery must be avoid (see the circle at the top left of Fig. 2, Pelagic 5);

- In the same way, when the biomass of pelagic fish is high, the value of demersal fish biomass must not be too low to guarantee a perennial system and some low levels of catch must be avoided (see the circle at the bottom of Fig. 2, Pelagic 60);
- For mean values of pelagic fish biomass, there is no restriction about the fisheries.

Fig. 3 presents a 2D slice of the viability kernel, when detritus biomass = 2000 tonnes/km^2, phytoplankton = 400 tonnes/km^2 and zooplankton = 130 tonnes/km^2. We note that the viability kernel is smaller: a high level of pelagic fish represents a non-viable situation. Again, some high and low levels of fisheries must be avoided. In general, when the value of zooplankton is higher, the viability kernel is smaller and there is no viable path starting from pelagic fish biomass = 60 tonnes/km^2.

4.2. *Effects of fisheries on pelagic fish*

We explore now the impact of fisheries on pelagic fish, keeping the same values for others species.

Fig. 4 presents the viability kernel where detritus biomass = 2000 tonnes/km^2, phytoplankton = 90 tonnes/km^2, zooplankton = 100 tonnes/km^2 and for demersal fish = 5, 15, 30 tonnes/km^2.

We notice that fisheries affect the boundary of the viability kernel only when the demersal fish biomass is too low: the more the pelagic biomass is, the more the catch can be important. However, whatever the level of demersal fish, the level of fisheries must be controlled to guarantee a healthy system. For mean values of demersal fish, the system is not viable for low and high values of pelagic fish. For high values of demersal fish, the pelagic biomass must not be too low to guarantee the persistence of the ecosystem.

For high values of zooplankton (see Fig. 5), the viability kernel is smaller: some values of demersal fish and fisheries are necessary to ensure a viable path:

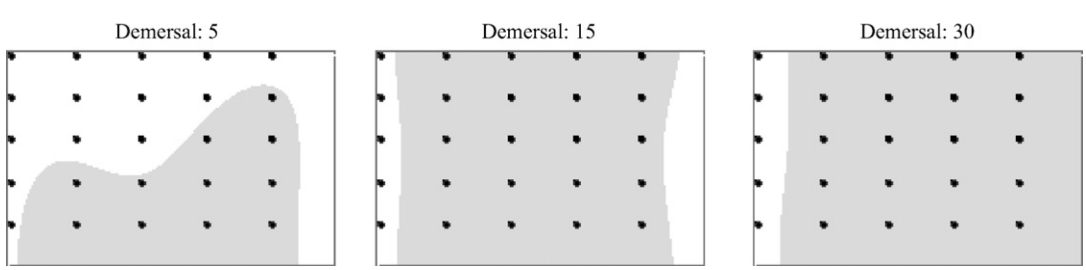

Fig. 4 – Approximation of viability kernel. The horizontal axis represents pelagic fish, vertical axis fisheries, detritus = 2000 tonnes/km^2, zooplankton = 90 tonnes/km^2 and phytoplankton = 100 tonnes/km^2.

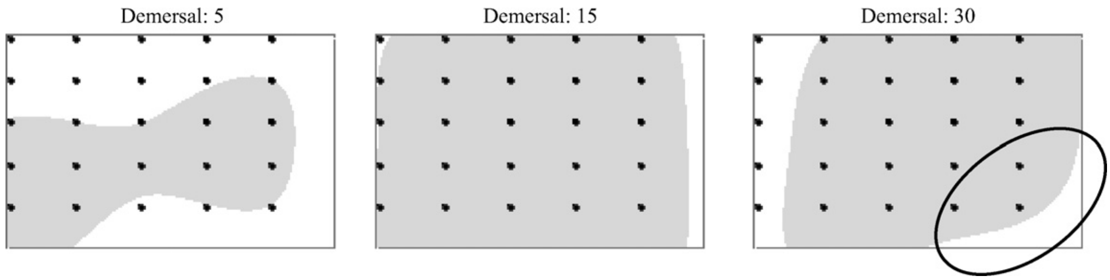

Fig. 5 – Approximation of viability kernel. The horizontal axis represents pelagic fish, vertical axis fisheries, detritus = 2000 tonnes/km^2, zooplankton = 130 tonnes/km^2 and phytoplankton = 400 tonnes/km^2.

- For low values of demersal fish, the level of fisheries must be carefully set; lower and higher values of catch represent non-viable situation;
- For mean value of demersal fish, the system is not viable for high values of pelagic fish;
- Fisheries have an influence for high biomass of demersal and pelagic fish: a minimum level of yields is necessary to ensure ecosystem persistence (area surrounded in Fig. 5).

4.3. *Main results*

Our study illustrates the potential utility of the viability kernel to help the definition of viable fishery policies: given values of the biomass of the five species, the viability kernel provides the levels of catch to avoid. In addition, the viability kernel defines some conditions in which the fisheries can be increased without compromising the viability. We notice that the maximum thresholds for fisheries used by Mullon et al. (2004) can also be increased.

5. Discussion and conclusion

Solving the viability problem provides all yield policies (if any) which guarantee a perennial system. This study shows that it is possible and interesting to integrate fisheries as a control parameter of a viability problem. We made strong simplifications: we supposed the same yield for the two species, and we should obviously take other parameters into account, like social and economics issues (Mullon et al., 2004). Nevertheless, we think that this work illustrates the potential of the viability approach to help the definition of fishery policies.

One of the main practical difficulties up to now with the viability theory was the lack of methods to solve the problem in a large number of dimensions. The use of learning procedures such as SVMs gives this theory a larger practical potential. However, to deal with a problem of six dimensions with the current algorithm can only be done with a very rough precision and several improvements are necessary to get more reliable and accurate results.

Moreover, it will be interesting to define yield strategies which allow the system to come from a non-viable state back to a viable state in minimum time, or minimizing some cost. This relates to the definition of the resilience proposed in (Martin, 2004).

Acknowledgements

The authors are grateful to I. Alvarez, J.P. Aubin, N. Bonneuil and P. Saint-Pierre for useful discussions.

Appendix A. Algorithm of SVM viability kernel approximation

We consider a given time interval dt and we define the set-value map $G: X \rightarrow X$

$$G(\mathbf{x}) = \{\mathbf{x} + \varphi(\mathbf{x}, \mathbf{u})\,\mathrm{d}t \text{ for } \mathbf{u} \in U(\mathbf{x})\}. \quad (8)$$

Considering the compact viability constraint set K, the viability kernel of K under G is the largest set included in K such that, for any $\mathbf{x}$ in Viab(K):

$$G(\mathbf{x}) \cap \mathrm{Viab}(K) \neq \emptyset. \quad (9)$$

We define a grid K_h as a finite set of K such that:

$$\forall \mathbf{x} \in K,\ \exists \mathbf{x}_h \in K_h \text{ such as } \|\mathbf{x} - \mathbf{x}_h\| < \beta(h). \quad (10)$$

At each step n, we define a discrete set $K_h^n \subset K_h^{n-1} \subset K_h$ and a continuous set $L(K_h^n)$ which is a generalization of the discrete set and which constitutes the current approximation of the viability kernel. The boundary of this set is define thanks to a particularly procedure, the support vector machines (SVM), which is a method for data classification. Given a set of examples $\{(\mathbf{x}_i, y_i)\}_{i=1}^n$ where $\mathbf{x}_i$ is a real vector and $y_i \in \{-1, 1\}$, SVM define a function f which separates examples of each labels:

$$f(\mathbf{x}) = \sum_{i=1}^{n} \alpha_i y_i k(\mathbf{x}_i, \mathbf{x}) + b \quad (11)$$

with $\alpha_i \geq 0$ and $k(\mathbf{x}_i, \mathbf{x}) = \exp(-\|\mathbf{x}_i - \mathbf{x}\|^2/2\sigma^2)$.

In Deffuant et al. (2007), we show that it is possible to find an optimal control vector $\mathbf{u}^*$, which defines the position the most inside the current approximation of the kernel among all possibilities in $G(\mathbf{x})$ (we use a gradient algorithm).

The steps of the algorithm are the following:

- Initialize the sets $K_h^0 = K_h$ and $L(K_h^0) = K$.
- Iterate:
  - Define the discrete set K_h^{n+1} from K_h^n and f_n as follows: $K_h^{n+1} = \{\mathbf{x}_h \in K_h^n \text{ such that } f_n(\mathbf{x}_h + \varphi(\mathbf{x}_h, \mathbf{u}^*)\,dt) \geq -1 \text{ and } (\mathbf{x}_h + \varphi(\mathbf{x}_h, \mathbf{u}^*)\,dt) \in K\}$.
  - If $K_h^{n+1} \neq K_h^n$ then run the SVM on the learning sample obtained with the points $\mathbf{x}_h$ of the grid K_h, associated with the labels +1 if $\mathbf{x}_h \in K_h^{n+1}$, and with labels -1 otherwise. Let f_{n+1} be the obtained classification function. $L(K_h^{n+1})$ is defined as follows: $L(K_h^{n+1}) = \{\mathbf{x} \in K \text{ such that } f_{n+1}(\mathbf{x}) = +1\}$.
  - Else stop and return $L(K_h^n)$.

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